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OR THE

LOGIC OF QUALITY.

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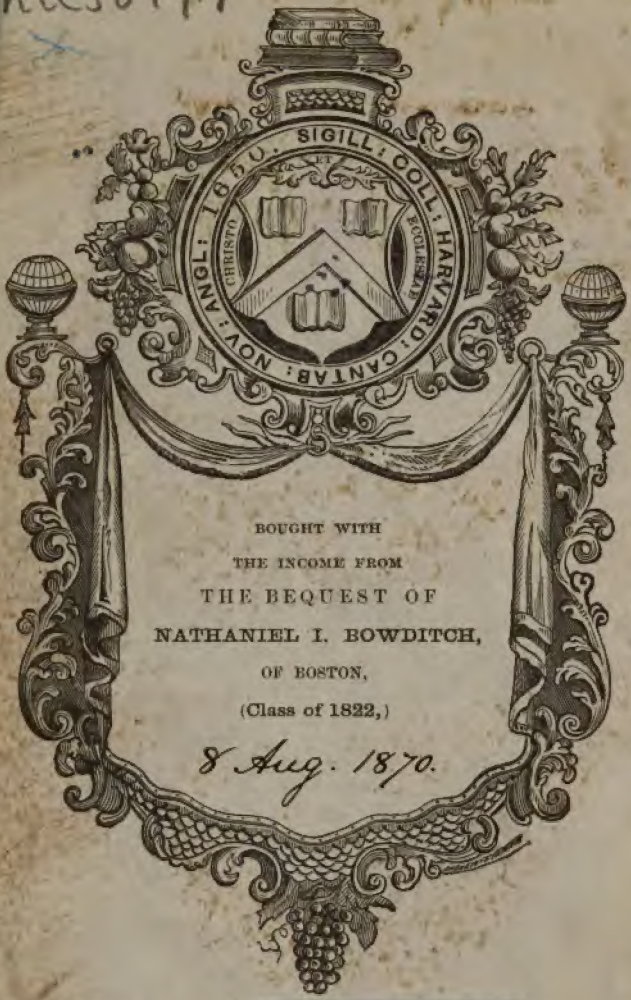

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OR

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PURE LOGIC

OR THE

LOGIC OF QUALITY APART FROM QUANTITY:

WITH

REMARKS ON BOOLE'S SYSTEM AND

ON THE RELATION OF LOGIC AND MATHEMATICS.

BY

William
W. STANLEY JEVONS, M.A.

Logica est ars artium et scientia scientiarum. — Scotus.

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PURE LOGIC.

INTRODUCTION.

It is the purpose of this work to show that Logic assumes a new degree of simplicity, precision, generality, and power, when comparison in quality is treated apart from any reference to quantity.

Extent and intent of meaning.

1. It is familiarly known to logicians that a term must be considered with respect both to the individual things it *denotes*, and the qualities, circumstances, or attributes it *connotes*, or implies as belonging to those things. The number of individuals denoted forms the breadth or *extent* of the meaning of the term; the qualities or attributes connoted form the depth, comprehension, or *intent*, of the meaning of the term. The extent and intent of meaning, however, are closely related, and in a reciprocal manner. The more numerous the qualities connoted by a term, the fewer in general the individuals which it can denote; the one dimension, so to speak, of the meaning being given, the other follows, and cannot be given or taken at will.

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Expression usually combined.

2. Logicians have generally thought that a proposition must express the relations of extent and intent of the terms at one and the same time, and as regarded in the same light. The systems of logic deduced from such a view, when compared with the system which may otherwise be had, seem to lack simplicity and generality.

Separation necessary.

3. It is here held that *a proposition expresses the result of a comparison and judgment of the sameness or difference of meaning of terms, either in intent or extent of meaning.* The judgment in the one dimension of meaning, however, is not independent of the judgment in the other dimension. It is only then judgment and reasoning in one dimension which is properly expressed in a simple system. Judgment and reasoning in the other dimension will be and must be implied. It may be expressed in a numerical or quantitative system corresponding to the qualitative system, but its expression in the same system destroys simplicity.

I do not wish to express any opinion here as to the nature of a system of logic in extent, nor as to its precise connection with the pure system of logic of quality.

Primary system.

4. Reasoning in quality and quantity, in intent or extent of meaning, being considered apart, it seems obvious that the comparison of things in quality, with respect to all their points of sameness and difference, gives the primary and most general system of reasoning. It even seems likely that such a system must comprehend all possible and conceivable kinds of reasoning, since it treats

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of any and every way in which things may be same or different. All reasoning is probably founded on the laws of sameness and difference which form the basis of the following system.

Present task.

5. My present task, however, is to show that *all and more than all the ordinary processes of logic may be combined in a system founded on comparison of quality only, without reference to logical quantity.*

Relation to Boole's Logic.

6. Before proceeding I have to acknowledge that in a considerable degree this system is founded on that of Prof. Boole, as stated in his admirable and highly original Mathematical Analysis of Logic.* The forms of my system may, in fact, be reached by divesting his system of a mathematical dress, which, to say the least, is not essential to it. The system being restored to its proper simplicity, it may be inferred, not that Logic is a part of Mathematics, as is almost implied in Prof. Boole's writings, but that the Mathematics are rather derivatives of Logic. All the interesting analogies or samenesses of logical and mathematical reasoning which may be pointed out, are surely reversed by making Logic the dependent of Mathematics.

* *Investigation of the Laws of Thought.* By George Boole, LL.D. London, 1854. Frequent reference will be made to this work in the following pages.

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CHAPTER I.

OF TERMS.

Of things and their names.

7. *Pure logic arises from a comparison of things as to their sameness or difference in any quality or circumstance whatever.*

In discourse we refer to things by the aid of marks, names, or terms, which are also, as it were, the handles by which the mind grasps and retains its thoughts about things. Thus correct thought about things becomes in discourse the correct use of names. Logic, while treating only of names, ascertaining the relations of sameness and difference of their meanings, treats indirectly, as alone it can, of the samenesses and differences of things.

Meaning of name.

8. A term taken in intent has for its meaning the whole infinite series of qualities and circumstances which a thing possesses. Of these qualities or circumstances some may be known and form the description or definition of the meaning; the infinite remainder are unknown.

Among the circumstances, indeed, of a thing, is the fact of its being denoted by a given name, but we may speak of a thing, of which only the name is known, as having *a name of unknown meaning.*

The meaning of every name, then, is either un-

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known or more or less known. But we may speak of a term that is more or less known as being simply *known*.

Qualities infinite in number.

9. Among the qualities and circumstances of a thing is to be counted everything that may be said of it, affirmatively or negatively. Any possible quality or circumstance that can be thought of either does or does not apply to any given thing, and therefore forms, either affirmatively or negatively, a quality or circumstance of the thing. Concerning anything, then, there may be an infinite number of statements made, or qualities predicated.

Relation of meanings.

10. When we assign a name to a thing, with knowledge of, and regard to, certain of its qualities or circumstances, that name is equally the name of anything else of exactly the same known qualities and circumstances. For there is nothing in the name to determine it to the one thing rather than the other. Any name, then, must be the name *in extent* of anything, and of all things agreeing in the qualities or circumstances which form its known meaning *in intent*, and in this system.

Present meaning.

11. Though it is well to point out that all our names or terms bear a universal quantity when regarded in extent, it must be understood, and constantly borne in mind, that further reference to the meaning of a term in extent or quantity of individuals, is excluded in these pages.

The primary and only present meaning of a name or term is a certain set of qualities, attributes, properties, or circumstances, of a thing unknown or partly known.

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Term defined.

12. *Term* will be used to mean *name*, or any combination of names and words describing the qualities and circumstances of a thing.

Generality of our terms.

13. The terms of this system may be made to express any combination of samenesses and differences in quality, kind, attribute, circumstance, number, magnitude, degree, quantity, opposition, or distance in time or space. A term may thus represent the qualities of a thing or person in all the complexity of real existence, so well and fully defined that we cannot suppose there are, or are likely to be, two things the same in so many circumstances. Such a term would correspond to the *singular*, *proper*, *non-attributive*, or *non-connotative* names of the old logic. Such names are accordingly by no means excluded from this system; and it is here held that the old distinction of *connotative* and *non-connotative* names is wholly erroneous and unfounded. If there is any distinction to be drawn, it is that singular, proper, or so-called non-connotative terms, are more full of connotation or meaning in intent or quality than others, instead of being devoid of such meaning.

Proper names.

Condition of sameness of meaning.

14. As logic only considers the relations of meaning of terms, as expressed within a piece of reasoning, the special meaning of any term is of no account, provided that *the same term have the same meaning throughout any one piece of reasoning.*

Thus, instead of the nouns and adjectives, to each of which a special meaning is assigned in common discourse, we shall use certain letters, A, B, C, D, ... U, V . . . each standing

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for a special term, or a definite meaning, and for *any term or meaning*, always under the above condition.

Terms known or unknown.

15. Our terms, A, B, C, ... like the terms of common discourse, may be either known or unknown in meaning. *It is the work of logic to show what relations of sameness and difference between unknown and known terms may make the unknown terms known.*

Were it not to explain *ignotum per ignotius*, we might say that *logic is the algebra of kind or quality, the calculus of known and unknown qualities*, as algebra (more strictly speaking *universal arithmetic*, which does not recognise essentially negative quantities) is the calculus of known and unknown quantities.

Symbols of plain meaning.

16. Let it be borne in mind that the letters A, B, C, &c., as well as the marks +, 0, and =, afterwards to be introduced, are in no way mysterious symbols. The term A, for instance, is merely a convenient abbreviation for any ordinary term of language, or set of terms, such as *Red*, or *the Lords Commissioners for executing the office of the Lord High Admiral of England.*

Again, + is merely a mark substituted for the sake of clearness, for the conjunctions *and*, *either*, *or*, &c., of common language. The mark = is merely the copula *is*, or *is same as*, or some equivalent. The meaning of 0, whatever it exactly be, may also be expressed in words. There is consequently nothing more symbolic or mysterious in this system than in common language.

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CHAPTER II.

OF PROPOSITIONS.

Proposition defined.

17. *A proposition is a statement of the sameness or difference of meaning of two terms*, that is, of the sameness or difference of the qualities and circumstances connoted by each term.

Affirmative, negative. Its purpose.

18. According as a proposition states *sameness* or *difference*, it is called *affirmative* or *negative*.

19. *It is the purpose or use of a proposition to make known the meaning of a term that is otherwise unknown.*

Truth and falsity.

20. A proposition is said to be true when the meanings of its terms are same or different, as stated; otherwise it is called false or untrue. As logic deals with things only through terms, it cannot ascertain whether a proposition is true or false, but only whether two or more propositions are or are not true together, under the condition of meaning of terms (§ 14).

Notation of affirmative proposition.

21. We denote by the copula *is*, or by the mark =, the sameness of meaning of the terms on the two sides of a proposition.

For the present we shall speak only of affirmative propositions, which are of superior importance; and when not otherwise specified, *proposition* may be taken to mean *affirmative proposition*.

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Conversion of propositions.

22. *A proposition is simply convertible.* The propositions A=B and B=A are the same statement; either of the terms A and B is the same in meaning as the other, undistinguishable except in name.

This simple conversion comprehends both the *simple conversion*, and *conversio per accidens* of the school logic.

One term known from one proposition.

23. *One proposition and one known term may make known one unknown term.*

From A=B, so far as we know B, that is, know its meaning, we can learn A; so far as we know A, we can learn B.

We thus know samely of both sides of a proposition whatever we know of either. The same might be said of uncertain or obscure knowledge.

Useless and identical propositions excluded.

24. A proposition between any two terms of which the meanings are otherwise known as same or different, is useless. For it cannot serve the purpose of a proposition (§ 19). Such is any proposition between a term and itself, as A=A, B=B (§ 14). These useless propositions are called *Identical*. They state the condition of all reasoning, but we know it without the statement.

A proposition repeated, or a converted proposition (§ 22), is also useless, except for the mere convenience of memory, or ready apprehension.

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CHAPTER III.

OF DIRECT INFERENCE.

Law of sameness.

25. It is in the nature of thought and things, that *things which are same as the same thing are the same as each other.*

More briefly—SAME AS SAME ARE SAME.

Hence the first law of logic—that *terms which are same in meaning as the same term, are the same in meaning as each other.*

This law, it is obvious, is analogous to Euclid's first axiom, or common notion, that *things which are equal to the same thing, are equal to each other.* Things are called equal which are *same in magnitude*, but what is true of such sameness, is also true of sameness in any way in which things may be same or different. Euclid's geometrical law is but one case of the general law.

Meaning of laws of logic.

26. Logic proceeds by laws, and is bound by them. For logic must treat names as thought treats things. And the laws of logic state certain *samenesses* or uniformities in our ways of thinking, and are of self-evident truth.

Direct inference shown and defined.

27. *When two affirmative propositions are same in one member of each, the other members may be stated to be same.*

From A=B, B=C, which are the same

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in the member B, we may form the new proposition A=C. For A and C being each stated to be the same as B, may by the law of sameness be stated to be the same as each other.

A proposition got by the Law of Sameness is said to be got by *direct inference*, and is called a *direct inferent*, or, in common language, *a direct inference.*

Premise defined.

28. Propositions from which an inference is drawn are called *premises*, and are given or taken as the basis of reasoning. Logic is not concerned with the truth or falsity of premises or inference, except as regards the truth or falsity of one with the other. (§§ 20, 37.)

Expression defined.

29. An *expression* for a term consists of any other term which by premises we know to be the same in meaning with that term.

Elimination defined.

30. In inferring a new proposition from two premises we are said to *eliminate* or remove the member which is the same in the two premises.

From two premises we may eliminate only one term, and infer one new proposition. By saying that *we may*, it is not meant that *we always can.*

Related propositions defined.

31. Propositions are said to be *related* to each other which have a same or common member, or which are so related to other propositions so related; and so on.

In other words, any two propositions are related which form part of a series or chain of propositions, in which each proposition is related to the adjoining ones or one.

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Related terms defined.

32. Terms are said to be *related* which occur in one same, or in any related propositions.

Use of syllogism.

33. *From two related premises and one known term we may learn two unknown terms, and not more.*

From A=B and B=C, we learn any two of A, B, C, when the third is known.

Series of premises.

34. *From any series of related premises, and one known term we may learn as many unknown terms as there are premises.* Thus, from A = B =C=D=E=F, we may learn any five terms when the sixth is known. For each useful proposition may render one unknown term known (§ 19). Between each two adjoining premises one term may be eliminated, becoming known in one premise, and rendering another term known in the other. There must at last remain a single proposition containing two terms, each of which occurs only in one premise.

Number of terms and related premises

35. *The number of related premises must be one less than the number of different terms.* If it be still less, the propositions cannot be all related; if it be greater, some of the premises must be useless, because they must lie between terms otherwise known to be same by inference.

It will be remarked that systems of mathematical propositions or equations with known and unknown qualities are perfectly analogous in their properties to logical propositions.

Irrelevant terms and premises.

36. When a related premise contains a term or member not relevant to the purpose of the reasoning, this term is eliminated by neglecting the premise; and for every such premise neglected

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a term is eliminated. In regard both to related and unrelated premises and their terms, the *neglect* of all irrelevant terms and premises may be considered a process of elimination which accompanies all thought.

37. *Inference is judgment of judgments, and ascertains the sameness of samenesses.* *Science of Science.*

When in comparing A with B, and the same B with C, we judge that A=B and B=C, we obtain *science*, or reasoned knowledge of things, as distinguished from the mere knowledge of sense or feeling. But when we judge the judgments A=B=C to be the same, as regards A and C, with the judgment A=C, we obtain *Science of Science.*

Here is the true province of logic, long called *Scientia Scientiarum.* Hence it is that logic is concerned not with the truth of propositions *per se* (§ 20), but only with the truth of one as depending on others.

SCIENCE OF SCIENCE	{A=B=C}	=	{A=C}	REASONING
SCIENCE	A=B		B=C	JUDGMENT
THINGS	A	B	C	APPREHENSION

38. Here we find the clear meaning of the distinction of *form and matter of thought.* *Form and matter.*

Sameness of Samenesses = Form
Sameness of things = Matter + Form
Things = Matter
} Of thought

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CHAPTER IV.

OF COMBINATION OF TERMS.

Addition of meanings.

39. In discourse, when several names are placed together side by side, the meaning of the joint term is sometimes the sum of the meanings of the separate terms.*

So in our system, *when two or more terms are placed together, the joint term must have as its meaning the sum of the meanings of the separate terms.* These must be thought of together and in one.

* I shall here consider only the cases of combination in which the combined term means the *added meanings* of the separate terms. The same forms of reasoning apply, as I believe, *mutatis mutandis*, to any cases of combination under some such wider law as this—

Same parts samely related make same wholes.

Only by some such extension can logic be made to embrace the major part of all ordinary reasoning, which has never yet been embraced by it, save so far as this may have been done in some of Professor De Morgan's latest writings. But to show how such an extension may be grafted on to my system must be reserved for a future opportunity. In most relations it is obvious that the order of terms in relation is no longer indifferent. (§ 41.)

Concerning some inferences by combination, see Thomson's *Outlines*, §§ 87, 88.

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40. Any terms placed together will be said to form with respect to any of those separate terms a *combination* or *combined term*. With respect to all other terms they may be called simply *a term*. For it must be remembered that any single term, A, B, C, &c., is not more single in meaning than a combination. *Combination defined.*

41. *The meaning of a combination of terms is the same in whatever order the terms be combined.* *Order of combination indifferent.*

Thus, AB = BA; ABCD = BACD = DCAB, and so on.

For the order of the terms can at most affect only the order in which we think of them, and in things themselves there is no such order of qualities and circumstances. (Boole, p. 30.)

42. *A combination of a term with itself is the same in meaning with the term alone.* *Law of simplicity.*

Thus AA = A, AAA = A, and so on.

Also, a combination of terms is not altered by combination with the whole or any part of itself. Thus ABCD=ABCD . BCD=A . BB . CC . DD =ABCD, since BB = B, CC = C, DD = D.

The coalescence of same terms in combination must be constantly before the reader's mind.

This important and self-evident law of logic was first brought into proper notice by Prof. Boole (p. 32), who remarks: 'To say "good, good," in relation to any subject, though a cumbrous and useless pleonasm, is the same as to say "good."'

Professor Boole gave to this law the name *Law of Duality*. But as this name, on the one hand is not peculiarly adapted to express the

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general fact AAAAA ..... = A, and is peculiarly adapted to express the fact A = AB + A*b* (§ 99), I have ventured to transfer the name, and substitute a new one.

Degree of quality.

43. In the terms as used under the above law there is no reference to *degree of quality*. When required, each degree of quality may be treated in a separate term, containing as part of its meaning every less degree of the quality. Two or more degrees of a same quality in logical combination therefore produce the greatest of those degrees.

Law of same parts and wholes.

44. It is in the nature of thought and things that *when same qualities are joined to same qualities the wholes are same.*

Hence the law of logic—

Same terms combined with same terms give same combined terms.

Thus, since A = A and B = B, therefore AB = BA = AB.

This self-evident law is a more general case of Euclid's second axiom. It may, perhaps, be most briefly stated as follows:—*Same parts make same wholes.*

Inference by combination.

45. *Same terms being combined with both members of a premise, the combinations may be stated as same in a new proposition which will be true with the premise.*

For what is true of terms obviously the same, as A, A, or B, B, must also be true of terms known to be the same in meaning by a premise. Thus, from A = B we may infer AC = BC by combining C with each of A and B.

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As the number of possible terms which may be combined with the terms of a premise is infinite, there may be drawn from any premise an infinite number of inferences by combination.

Combination of propositions.

46. Inferences which may be drawn by combining the members of two or more premises need not be considered here.

General truth of such inferences.

47. A proposition inferred by combination (§ 45) will be true with its premise, whatever be the term or terms used for combination. When terms of specific meaning, indeed, are selected at random, it will usually happen that the combinations of the inference are unheard-of, absurd, and useless. This does not affect the truth of the inferred proposition, which only asserts that the meaning of the one combination, whatever it be, is the same as the meaning of the other.

Tacit relations excluded.

48. In our daily use of specific terms, we constantly use each under the restriction of a number of premises so well known to all persons that it is needless to express them. Terms joined not in accordance with these tacit relations make nonsense. For instance, the impassable difference of matter and mind renders it nonsense to join the name of any material with that of any mental attribute, except in a merely metaphorical sense. In order, then, that our inferences should always be intelligible and useful, we should require the expression of all tacit premises connected with terms of specific meaning. It is only the several branches of science, however, that can undertake the necessary investigations in detail. Our inference remains true, however complicated be the

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relations of sameness and difference of the terms introduced. But it is inference from premises which *are stated*, not from those which *might be* or *ought to be stated*.

Formation of common term.

49. *When premises contain terms only partially the same, the combination of each with the part that is different in the other will produce a term completely the same in each.* Such premises may be considered as *related*. (§ 31.)

Thus, in A = C and B = CD, the terms C and CD are only partially the same. But the combination of D with A = C gives AD = CD, having one member completely the same as one member of B = CD. Hence we may infer AD = CD = B (§ 26), and eliminate the term C, which was common in the premises: thus, AD = B.

Again, to eliminate B from the premises A = BC and E = BD, combine D with each side of the first, and C with each side of the second. Hence, AD = BCD = CE, or AD = CE, in which B does not appear.

Useless cases.

50. From premises which have no term in common, this process will only give us the inferences which might be had (§ 46) by the direct combination of the respective terms of the premises. Thus, A = B and C = D give AD = BD, and BC = BD, whence AD = BC. And we might similarly get AC = BD.

Substitution defined.

51. The following process may be called *substitution*, and will be seen to give the same inference as the two processes of forming a common term (§§ 49, 27), and then eliminating it.

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For any term, or part-term, in one premise, may be substituted its expression (§ 29) *in other terms.*

In short, the two members of any premise may be used indifferently, one in place of the other, wherever either occurs.

Thus, if A=BCD and BC=E, we may in the former premise substitute for BC its expression E, getting A=DE. The full process of inference consists in combining D with both sides of BC=E, and eliminating the complete common term BCD thus obtained, so that A=BCD=DE.

Intrinsic elimination.

52. *We may substitute for any part of one member of a proposition the whole of the other.*

Thus, in A=BCD, we may substitute for any one of B, C, D, BC, BD and CD, parts of BCD the one member, the whole, A, of the other member, inferring the new propositions—

A=ACD	A=ABD	A=ABC
A=AD	A=AB	A=AC.

The validity of this process depends on the Laws of Simplicity (§ 42), and of Part and Whole (§ 44), as is seen by combining each member of the premise with itself. Thus, from A=BCD we have A.A=BCD.BCD=BCD.D =AD by coalescence of same terms, and substitution for BCD of its expression A.

The new proposition thus inferred will have one of its sides *pleonastic*, that is, with some part of its meaning repeated. But it is obvious that we cannot, as a general rule, substitute for part of one side less than the whole of the other, because

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we cannot from the premise alone know that the meaning of the part-term removed is quite supplied in the part of the other member put for it.

The above process may be called *intrinsic elimination*, to distinguish it from the former process of elimination between two premises, which may be called *extrinsic elimination*, and is seen to be that case of intrinsic elimination in which we substitute for the whole of one side the whole of the other. In a single premise, intrinsic elimination of a whole member would give only an identical and useless result.

Intrinsic elimination gives no new knowledge, but is of constant use in striking out or *abstracting* terms concerning which we do not desire knowledge, and which are therefore worse than useless in our results.

Professor Boole's system of elimination (p. 99), is, I believe, equivalent to the above, though the correspondence may not at first sight be apparent.

Failure of elimination.

53. A term cannot be intrinsically eliminated which occurs in both members of a proposition. The presence of such part-term may be called a *condition* of the sameness of the remainder of the terms.

Samely related terms.

54. Terms are said to be samely related in a premise when their interchange does not alter the premise.

Thus, B and C are samely related in $A=BC$, because the premise is the same $A=CB$ (§ 41) after their interchange. But A and B are not samely related, because their interchange alters the premise into $B=AC$.

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In A=BCDE . . . any two of B, C, D . . . . are samely related and may be interchanged.

55. Of samely related terms, an expression for the one is the same as the expression for another after the two terms in question have been interchanged. *Inference concerning two such.*

56. When several terms are samely related, we obtain the expressions concerning the rest from the expression for any one by successively changing each term into the next when the terms are kept in some fixed order. *Concerning several.*

It is evident that we may always interchange the terms in any part of a problem, provided we do so throughout the problem (§ 14). And in those cases in which the premises remain unchanged thereby, we evidently get several inferences from the same premises. This method of interchanges is familiar to mathematicians.

57. It will be obvious that a mathematical term or quantity of several factors is strictly analogous in its laws to a logical combined term, *excluding the Law of Simplicity.* *Mathematical analogy.*

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CHAPTER V.

OF SEPARATION OF TERMS.

Law of same wholes and parts.

58. It is in the nature of thought and things that *when from same sets of qualities same qualities are taken, the remaining sets are the same;* or, more briefly—*Same parts from same wholes leave same parts.*

Hence the logical law:—*When from same combinations of terms same terms are taken, the remaining terms are the same.*

This is the converse of the Law of Same Parts and Wholes (§ 44), and is equally self-evident with it. But it is not equally useful with it; and in Pure Logic, in fact, is of no use at all. The removal of *terms* with their known meanings is not equally possible with their combination, and in useful logical premises, is not possible at all. For, in a useful premise (§ 19), a part at least of one member must be unknown, and this part may or may not contain the part we desire to remove. Even supposing then that a term occurs on either side of a premise, we cannot remove it from the known side, because we cannot know whether or not we can remove it from the unknown or partially known side.

Thus in AB=AC, suppose A and C known,

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and B unknown. We cannot infer B=C, because B may contain part or the whole of the known meaning of A, in addition to the known meaning of C, by the Law of Simplicity (§ 42), and in leaving B, we do not remove A from one member of the premise.

Complete analogy of logic and mathematics.

59. The logic of known and unknown terms, it has been said (§ 15), is analogous to the calculus of known and unknown numbers.

So, a logic in which all terms were known would have an analogue in common Arithmetic, a calculus in which all the numbers employed are known. Combination of terms has an analogue in multiplication of numbers, and separation of terms in division of numbers. As in logic combination is unrestricted, so in calculus is multiplication. As in logic of known terms only, separation of terms is unrestricted, so in a calculus of known numbers only, division is unrestricted. But, *as in logic of known and unknown terms separation is restricted, so in calculus of known and unknown numbers division is restricted.*

Restriction of division.

60. It is well known that, in like manner, we cannot divide both sides of an equation by an unknown factor, and assert the resulting equation to be necessarily true, because the unknown factor may be $= 0$. Thus, from $xy = xz$, we cannot remove x, and assert $y = z$, because if x happen to be $= 0$, the equation $xy = xz$ is true, whatever finite numbers be the meanings of y and z.

The correspondence is thus shown :—

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Logical Propositions.	Mathematical Equations.
Terms known admit	*Numbers known admit*
Combination	Multiplication
Separation (unless either dividend contain divisor)	Division (unless divisor = 0)
Terms unknown admit	*Numbers unknown admit*
Combination *but do not admit*	Multiplication *but do not admit*
Separation	Division

The above analogies did not escape the notice of Professor Boole (pp. 36-37), and I am therefore at a loss to understand on what ground he asserts that there is a breach in the correspondence of the laws of logic and mathematics.

Parts not known from the whole.

61. *From the meaning of a whole term we cannot learn the meaning of a part.*

In A=BC, if we know A we learn BC as a whole; but we do not thence learn the parts B, C, separately. For of the qualities in A any part may be in B, and any part in C, including any part of those in B, by the Law of Simplicity (§ 42). It is only necessary that every quality in A shall be either in B or in C. Even if we know one of B and C, we only learn of the other that it must contain any quality of A not in the first.

We here meet the imperfection of an inverse process.

Number of terms and premises.

62. With reference to the relation between the number of premises, and the numbers of known and unknown terms (§§ 33-35), we must treat as separate terms any which occur separate in premises, although they may also occur in combination. Otherwise, we always treat any whole combination as a single term.

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CHAPTER VI.

OF PLURAL TERMS.

63. *A plural term has one of several meanings, but it is not known which.* *Terms of many meanings.*

Thus *B or C* is a plural term, or term of many meanings, for its meaning is either that of B or that of C, but it is not known which.

A term not in form plural, may be distinguished as *single*; such is A.

64. The separate terms expressing the several possible meanings of a plural term are called *alternatives*, and are to be joined together by the sign + placed between each two adjoining terms. *Alternative defined.*

All that has been said of single terms applies to plural terms, *mutatis mutandis.*

65. *The meaning of a plural term is the same whatever be the order of the alternatives.* *Order of alternatives.*

Either B or C is the same in meaning as *either C or B*, that is, $B+C=C+B$. For the order in which we think of the possible qualities of a thing cannot alter those qualities, and the order must not convey any intimation that one meaning is more probable than another.

66. *A term is combined with a plural term by combining it with each of its alternatives.* *Combination with plural term.*

For what is A and either B or C, if it is B,

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is AB; if it is C, is AC, and it is therefore either AB or AC.

Use of brackets.

67. Let a plural term enclosed in brackets (. . . . .), and placed beside another term, mean that it is combined with it, as one single term is with another:

Thus $A(B+C) = AB+AC$.

Combination of plural terms.

68. One plural term is combined with another by combining each alternative of the one separately with each of the other. Each combined alternative may then be combined with each alternative of a third plural term, and so on:

Thus $(D+E)(B+C)=B(D+E)+C(D+E)$
$=BD+BE+CD+CE.$

Law of unity.

69. It is in the nature of thought and things that *same alternatives are together same in meaning, as any one taken singly.*

Thus, what is the same as A *or* A is the same as A, a self-evident truth.

$A+A=A \quad A+A+A=A \quad A+A+B=A+B$

This law is correlative to the Law of Simplicity, (§ 39), and is perhaps of equal importance and frequent use. It was not recognised by Professor Boole, when laying down the principles of his system.

Superfluous terms.

70. In a plural term, any alternative may be removed, of which a part forms another alternative.

Thus the term *either B or BC* is the same in meaning with B alone, or $B+BC=B$. For it is a self-evident truth (§ 99) that B standing alone is either the same as BC, or as *B not-C*. Thus

$B+BC=B$ *not*-$C+BC+BC$
$=B$ *not*-$C+BC=B$.

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71. A plural term obeys the Law of Simplicity (§ 42). *Plural terms obey laws of single terms.*

For let $A=B+C$; then —

$$AA=(B+C)(B+C).$$
$$AA=BB+BC+BC+CC \quad (\S\ 68).$$
$$A=\ B+BC+C \quad (\S\ 42).$$
$$A=\ B+C \quad (\S\ 70).$$

A plural term obeys the Law of Unity (§ 69):

$$A+A=B+C+B+C=B+C.$$

72. For any alternative or part of an alternative may be substituted (§ 51) its expression in other terms: *Substitution in plural terms.*

Thus, if $A=B+CD$ and $D=E$, substitute, getting $A=B+CE$.

73. A plural term may be substituted like a single term for any term, single or plural, of which it is the expression. When in combination, the several alternatives must be separately combined (§§ 66, 68). *Substitution of plural terms.*

Conversely, for a plural term may be substituted its expression in a single term:

Thus, if $A=BC$ and $C=D+E$, for C substitute $D+E$, and $A=B(D+E)=BD+BE$.

Or from the premises $A=BD+BE=B(D+E)$ and $C=D+E$, we might by substitution get back to $A=BC$.

74. *A plural term is known when each of its alternatives is known.* *Plural term known.*

Thus, in $A=B+C$, A is known when the meanings of each of B and C are known. But of course from knowing a single meaning of A, we cannot learn either or both of B and C.

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Number of terms and premises.

75. With reference to the relation between the number of premises, and the numbers of known and unknown terms (§§ 33–35), we must treat as a separate term each alternative of a plural term.

A proposition with a plural term thus corresponds to an equation with several unknown quantities.

Plural and single terms.

76. As plural terms obey the laws of single terms, and a term single in form may be plural in meaning, it will not be necessary for the future to distinguish *plural and single terms*, any more than it has been to distinguish *combined and simple terms.*

There is some danger of misconception concerning plural terms. Though a plural term has one of several meanings, it cannot bear in this system more than one at the same time, so to speak. Hence it still remains a *unit*, the name of a single set of qualities, one of several sets, but it is not known which. The whole of this system in short is *unitary*, and involves the same remarkable analogies to a calculus of unity and 0 which have been brought forward so explicitly in Professor Boole's system.

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CHAPTER VII.

OF NEGATIVE PROPOSITIONS.

Terms may also be known and stated as differing, or not being the same in meaning.

Law of difference.

77. It is in the nature of thought and things that *a thing which differs from another differs from everything the same as that other.*

More briefly stated—*Same as different are different.*

Hence in logic—

A term which differs from another term in meaning differs from every term which is the same as that other.

If A is not the same as B, which is the same as C, then A is not the same as C. *The inference arises in the sameness of B and C, allowing us to substitute one for the other.* Hence we learn nothing of the sameness or difference of any two terms, D and E, each of which differs from a third, F; for D and E may each have any of an indefinite variety of meanings, and each may yet differ from F. (§ 152.)

Negative inference.

78. Hence a chain of related premises between any of which inferences can be drawn, must not contain more than a single negative premise.

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Also any inference in which a negative premise is concerned must be a negative inference.

Conversion.

79. *A negative proposition is simply convertible.* For *A is not the same as B*, is the same statement as *B is not the same as A*.

Law of different parts and wholes.

80. *When same terms are combined with different terms, the wholes may be different.*

If A differs from B, then AC differs from BC, provided, however, that the difference of A and B does not consist in any part of C.

Law of different wholes.

81. *When from different wholes same parts are taken, the remainders are different.*

This is equally self-evident with the preceding converse.

It is unnecessary further to consider negative propositions, because their inferences may be obtained by use of affirmative propositions.

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CHAPTER VIII.

OF CONTRARY TERMS.

Negative term.

82. *The known meaning of a negative term is the absence of the quality, or set of qualities, which forms the known meaning of a certain other, its positive term.*

Thus *not-A* is the negative term signifying the absence of the quality or set of qualities *A*. If the known meaning of *A* be only a single quality, *not-A* means its absence; but if *A* mean several qualities, *not-A* means the absence of any one or more.

Thus, if $A=B.C$

$not\text{-}A = B\ not\text{-}C + not\text{-}B.C + not\text{-}B\ not\text{-}C.$

Negative of negative.

83. *The negative of a negative term is the corresponding positive term.*

What is *not-not-A* is *A*.

Simple contrary terms defined.

84. Since the relation of a positive to a negative term is the same as the relation of a negative to a positive, let each be called the *simple contrary term* of the other.

Notation.

85. For convenience let *not-A* be written *a*. Then any large and its small letter denote a pair of simple contraries; and *not-a* is A. Also, the contrary of BC (§ 82) is

$$Bc + bC + bc,$$

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which expresses the absence of one or more of B and C.

Laws obeyed.

86. All that has been said of a term applies samely to one as to the other of a pair of contraries.

Thus, a obeys the several laws:

$$aa=a \quad a+a=a \quad \begin{matrix} C=D \\ aC=aD \end{matrix} \text{ and so on.}$$

'Involve' defined.

87. Let a combined term or a proposition be said to *involve* a term when it contains either that term or its contrary.

Contrary of plural term.

88. The contrary of a plural term is a term containing a contrary of each alternative.

Thus the contrary of $A+B+C$ is abc. If any alternative has more than one contrary, for each there will be a contrary alternative. Thus, $A+BC$ has the plural contrary $aBc+abC+abc$.

Contrary combinations.

89. Any combined term which contains the simple contrary of another term may be called *a contrary*, or *contrary combination* of this, or of any combination containing this.

Thus, any combined term containing A is a contrary of any term containing a, and it will seldom be necessary to distinguish by name *simple contraries*, such as A and a from contraries, or contrary combinations in general, which merely contain A or a. (See, however, §§ 99, 100.)

Law of contradiction.

90. It is in the nature of thought and things that *a thing cannot both have and not have the same quality.*

Contradictory term defined.

91. Hence a term which means a collection of qualities in which the same quality both *is and is not*, cannot mean the qualities of anything which is or ever will be known.

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Such a term then has *no meaning*, that is to say, no possible, useful, or thinkable meaning; but it may be said to mean *nothing*. Let it be called a *self-contradictory*, or, for sake of brevity, a *contradictory term*.

Use of 0.

92. Let us denote by the term or mark 0, combined with any term, that this is contradictory, and thus excluded from thought. Then $Aa=Aa.0$, $Bb=Bb.0$, and so on. For brevity we may write $Aa=0$, $Bb=0$. Such propositions are tacit premises of all reasoning.

Any two contrary terms in combination give a contradictory term.

Combination with contradictory.

93. Any term being combined with a contradictory, the whole is contradictory.

For the whole then means a collection of qualities which does and does not contain some same quality, and is therefore by definition a contradictory.

Thus, if $A=Bb=Bb.0$

$$AC=BbC=BbC.0.$$

Term 0.

94. The term 0, meaning *excluded from thought*, obeys the laws of terms.

$$0.0=0 \quad 0+0=0,$$

otherwise expressed:—What is excluded *and* excluded is excluded—What is excluded *or* excluded is excluded.

Condition of non-contradiction.

95. *Any term not known to be contradictory must be taken as not contradictory.*

Any term known to be contradictory is excluded from notice, and any term concerning which we are desiring knowledge must therefore be assumed not contradictory.

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Contradictory alternatives.

96. *In a plural term of which not all the alternatives are contradictory, the contradictory alternative or alternatives must be excluded from notice.*

If for instance $A=0+B$, we may infer $A=B$, because A if it be 0 is excluded; and if it be such as we can desire knowledge of, it must be the other alternative B.

Elimination of contradictory.

97. *No contradictory term is to be eliminated in direct inference.*

For all we can require to know of a contradictory term is that it is contradictory, and elimination of a contradictory term would prevent rather than give such knowledge.

Thus if $A=Cc\,.\,0$, $B=Cc\,.\,0$, all that we can require to know of A and B is known from these premises, and cannot be known from the inference $A=B$ got by eliminating the contradictory $Cc.\ 0$.

So, if $A=B=C=D=E=F=Gg\,.\,0$, the only useful inferences are those showing each of A, B, C, D, E, F, to be contradictory.

So, also, obviously, of intrinsic elimination.

It may be said, in fact, that contradiction supersedes all other elimination by itself eliminating all contradictory terms from further notice.

Elimination of alternatives.

98. An alternative is eliminated when its plural term is combined with a contrary of that alternative.

Thus, the alternative Ab is removed from the plural term $AB+Ab$ when combined with B.

$$(AB+Ab)B = AB + ABb = AB + 0 = AB$$

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Let $C = AB + Ab + aB + ab$

Then

$$AC = AB + Ab \qquad ABC = AB$$
$$BC = AB + aB \qquad AbC = Ab$$
$$aC = aB + ab \qquad aBC = aB$$
$$bC = Ab + ab \qquad abC = ab.$$

The term thus combined with each side cannot be eliminated intrinsically (§ 53), and remains *a condition of the rejection of the other alternative.*

It is by this rejection of alternatives that the extent or width of the meaning of a term is reduced, as its intent of known meaning is increased, by combination (§ 1). For every general term, in addition to its known meaning, may be assumed to have an indefinite multitude of unknown alternatives. In combination with a new term many of these will probably become contradictory.

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CHAPTER IX.

OF CONTRARY ALTERNATIVES.

Law of duality.

99. It is in the nature of thought and things that *a thing is either the same or not the same as another thing*. Otherwise—

A set of qualities either does or does not contain a certain quality.

Hence, in logic, a term must contain the meaning of one of any pair of *simple contrary terms*. Thus:—

A term is not altered in meaning by combination with any simple contrary terms as alternatives.

$$A = A(B + b) = AB + Ab.$$

For if A has meanings containing only B, then Ab is contradictory, and $A = AB + 0 = AB$.

If A has meanings containing only b, then $AB = 0$ and $A = 0 + Ab = Ab$.

If A has meanings of which some contain B and some b, the law is still true.

This Law of Duality is not the same as Professor Boole's law of duality. (See § 42.)

Apparent exceptions.

100. Some apparent exceptions may occur to this law. For instance, let A=virtue, B=black, and b=not-black. Then the statement

Virtue is either black or not-black, seems true according to the above law, and yet absurd. This

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arises from B and b not being simple contraries; for B may be decomposed into *black-coloured*—say BC, and b into *not-black-coloured*, or *not black and not coloured*, or $bC+bc$. Now, virtue is really not coloured at all, or is Abc, and, therefore, neither BC nor bC. Here, again, we must observe that the combination Bc is contradictory from the tacit premise *black is a colour* (§ 48).

Other apparent inconsistencies may be similarly explained.

Professor De Morgan has excellently said,* 'It is not for human reason to say what are the simple attributes into which an attribute may be decomposed.' And for such a reason it is that I have as far as possible abstained from treating any term as *known to be simple.*

Developement defined.

101. Let a term, combined with simple contraries as alternatives, be called a *developement* of the term as regards the contraries.

Thus, $AB+Ab$ is called a developement of A as regards B, or in terms of B, or involving B.

Continued developement.

102. *Any term is same in meaning after combination with all the possible combinations of other terms, and their contraries as alternatives.*

Since $A=AB+Ab$, and, again, $A=AC+Ac$, we may substitute for A in $AB+Ab$ (§ 51) its expression in terms of C. Thus,

$$A=ABC+ABc+AbC+Abc.$$

Again, since $A=AD+Ad$, we may substitute a second time, getting

$A=ABCD+ABCd+\ldots\ldots+Abcd$, and so on.

* *Syllabus*, p. 60.

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Dual term defined.

103. Let any two alternatives, differing only by a single part-term and its contrary, be called a *dual term.*

Thus, $AB+Ab$ is a dual term as regards B, and $ABC+ABc$ as regards C, and we may speak of $B+b$ or $C+c$ as the *dual part.*

Reduction of dual term.

104. *A dual term may always be reduced to a single term by removal of the contrary terms, without altering the meaning.*

For the term thus obtained is, by the Law of Duality, the same in meaning as the former dual term (§ 99).

Thus, from such a term as $AB+Ab$, we may always remove the dual part $B+b$, and the meaning of the term A will still be as before, since $A=AB+Ab$ is a self-evident (§ 99) truth always in our knowledge.

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CHAPTER X.

OF CONTRARY TERMS IN PROPOSITIONS.

Affirmative into negative proposition.

105. *From any affirmative premise we may infer a negative proposition by changing any term on one side only into its contrary.*

From A=B we have A not=b; for evidently B is not=b, and hence, by Law of Difference (§ 77), A=B not=b, or A not=b.

From AB=AC, similarly, AB not=Ac.

Terms of negative proposition.

106. *The two terms of a negative proposition are contraries.*

For the two terms of a negative proposition are different in meaning. Hence there must be some quality or qualities in the meaning of one, and not in that of the other; thus, the combination of the two terms would mean both the absence and presence of a certain quality or qualities, and would be a contradictory. The two terms then are contrary (§ 89).

Negative into affirmative proposition.

107. *A negative proposition may be changed into an affirmative, of which one term is a term of the negative, and the other term this term combined with the contrary of the other term of the negative.*

Thus, if A not=B, then A=Ab; or, again, B=aB.

For developing A in terms of B (§ 101), we

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have $A=AB+Ab$, but A and B being contraries (§ 106), AB is contradictory or 0. Hence, $A=0+Ab=Ab$ (§ 96).

Similarly, we may show $B=0+aB=aB$. So, if AB not=AC, then $AB=ABc$. For $AB=ABC+ABc=ABc$, since ABC is contradictory. And we see that

$$ABc=AB \text{ (contrary of AC)}$$
$$=AB\,(Ac+aC+ac)=ABc+0+0.$$

Since we may now convert any negative proposition into an affirmative, it will not be further necessary to use negative propositions in the process of inference. (§ 81.)

Inference of negative propositions.

108. *From any contradictory combination we may infer that any part of the combination not itself contradictory is not the same in meaning as the remainder or any greater part.* That the two parts differ may be expressed in a negative proposition, or its corresponding affirmative.

For if the other part be contradictory, it cannot be the same as the first part, which is not contradictory. And if neither of the parts is contradictory in itself, they cannot be same in meaning, else their combination would not produce a contradiction.

The affirmative inferences corresponding (§ 107) to the negative ones deduced under this rule may be otherwise had, so that it seems unnecessary to consider the negative inferences further in this place.

List of laws.

109. The following are the chief laws or conditions of logic :—

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Condition or postulate. The meaning of a term must be same throughout any piece of reasoning; so that A=A, B=B and so on. (§ 14.)

Law of Sameness. (§ 25.)

A=B=C; hence A=C.

Law of Simplicity. (§ 42.)

AA=A, BBB=B, and so on.

Law of Same Parts and Wholes. (§ 44.)

A=B; hence AC=BC.

Law of Unity. (§ 69.)

A+A=A, B+B+B=B, and so on.

Law of Contradiction. (§ 90.)

Aa=0, ABb=0, and so on.

Law of Duality. (§ 99.)

A=A (B+b) =AB+Ab

A=A (B+b) (C+c)

=ABC+ABc+AbC+Abc and so on.

It seems likely that these are the primary and sufficient laws of thought, and others only corollaries of them. Logic may treat only of *known samenesses of things;* and differences of things need be noticed, only for the exclusion from pure logical thought of all that is self-contradictory.

In pure number and its science, on the other hand, differences of things only are noticed.

The Laws of Simplicity, Unity, Contradiction and Duality furnish the universal premises of reasoning. The Law of Sameness is of altogether a higher order, involving Inference, or the Judgment of Judgments.

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CHAPTER XI.

OF INDIRECT INFERENCE.

Use of developement.

110. Taken by itself, the developement of a term (§ 101) gives us no new knowledge about it. But taken with the premises of a problem, we may learn that some of the alternatives of the developement are contradictory and to be rejected. The remaining alternatives then form a new and often useful expression for the term.

Indirect inference.

111. In thus using a developement we are said to infer *indirectly*, because we use the premise to show what a term is, not directly by the Law of Sameness, but *indirectly by showing what it is not.*

Indirect Inference is direct inference with the aid of self-evident premises derived from the Laws of Contradiction and Duality. But all Inference is still by the Law of Sameness.

Example.

112. Let A=B: required expressions for A, B, a, b, inferred from this premise. Develope these terms as follows (§ 101):—

$$\mathrm{A}=\mathrm{AB}+\mathrm{A}b \qquad \mathrm{B}=\mathrm{AB}+a\mathrm{B}$$
$$a=a\mathrm{B}+ab \qquad b=\mathrm{A}b+ab$$

Examine which of the alternatives AB, Ab, aB, ab, are contradictory according to the premise A=B.

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A	combined	with	A=B	gives	A=AB
B	,,	,,	,,	,,	AB=B
a	,,	,,	,,	,,	$Aa=aB=0$
b	,,	,,	,,	,,	$Ab=Bb=0$

Hence we learn that aB and Ab are contradictory, and may be rejected, and that AB is not contradictory according to the premise. Of ab, which is not found among any of the above terms, we can learn nothing from the premise, and it therefore cannot be known to be contradictory. Striking out aB and Ab in the developements of A, B, a, b, we have—

$$A=AB+0=AB \qquad B=AB+0=AB$$
$$a=0+ab=ab \qquad b=0+ab=ab$$

Inferences.

113. We have here the two inferences A=AB B=AB which might have been had from the premise by combination (§ 45), and from which we may pass back by elimination of AB to the premise.

We also have $a=ab$, and $b=ab$, *which could not have been had by direct inference.* And by eliminating ab between these two we have the new inference $a=b$. This result, indeed, that *from the sameness of meaning of two terms, we may infer the sameness of meaning of their simple contraries* is evidently true.

Inference from many premises.

114. By a similar method we may draw inferences from any number of premises, namely, by developing any required term in respect of other terms, and striking out the combinations which are shown to be contradictory in any premise.

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Thus, from A=B and B=C, to infer expressions for A and *a*, we develope these terms as follows:—

$$A = ABC + ABc + AbC + Abc$$
$$a = aBC + aBc + abC + abc$$

By combination we then, when possible, render one side of each premise same with each of the alternative combinations, and learn from the other side whether the combination is known to be contradictory by the premise. All the combinations in the above developements will be found contradictory, except ABC and *abc*, and we thus get the inferences A=ABC, and *a*=*abc*, of which the former indeed might have been got directly.

Method of indirect inference.

115. The process of indirect inference may similarly be applied to drawing any possible inference or expression from any series of premises, however numerous and complicated. The full process may be abbreviated according to the following series of rules, which may be said to form THE METHOD OF INDIRECT INFERENCE:—

Developement.

1. Any premises being given, form a combination containing every term involved therein (§ 87). Change successively each simple term of this into its contrary, so as to form all the possible combinations of the simple terms and their contraries.

Comparison.

2. Combine successively each such combination with both members of a premise. When the combination forms a contradiction with neither side of a premise, call it an *included subject* of the premise; when it forms a contradiction with both sides, call it an *excluded subject* of the premise;

Included subject.

Excluded.

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when it forms a contradiction with one side only, call it a *contradictory combination* or *subject*, and strike it out. *Contradiction.*

We may call either an included or excluded subject a *possible subject*, as distinguished from a contradictory combination or *impossible subject*. *Possible.* *Impossible.*

3. Perform the same process with each premise. Then a combination is an included subject of a series of premises, when it is an included subject of any one; it is a contradictory subject when it is a contradictory of any one; it is an excluded subject when it is an excluded subject of *every* premise. *Repeated comparison.*

4. The expression for any term involved in the premises consists of all the included and excluded subjects containing the term, treated as alternatives. *Selection.*

5. Such expression may be simplified by reducing all dual terms (§ 104), and by intrinsic elimination (§ 52) of all terms not required in the expression. *Reduction.*

6. When it is observed that the expression of a term contains a combination which would not occur in the expression of any contrary of that term, we may eliminate the part of the combination common to the term and its expression. (See below, § 117.) *Elimination.*

7. Unless each term of the premises and the contrary of each appear in one or other of the possible subjects, the premises must be deemed inconsistent or contradictory. Hence there must always remain at least two possible subjects. (§ 159.) *Contradictory premises.*

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Example. **116.** Required by the above process the inferences of the premise A=BC.

Developement. The possible combinations of the terms A,B,C, and their contraries, are as given in the margin.

Comparison. Each of these being combined with both sides of the premise, we have the following results:—

ABC	ABC =ABC	ABC included subject
ABc	ABc =ABCc =0	ABc contradiction
AbC	AbC =ABbC =0	AbC contradiction
Abc	Abc =ABbCc=0	Abc contradiction
aBC	0=AaBC=aBC	aBC contradiction
aBc	0=AaBc =aBCc =0	aBc excluded subject
abC	0=AabC =aBbC =0	abC excluded subject
abc	0=Aabc =aBbCc =0	abc excluded subject

It appears, then, that the four combinations ABc to aBC are to be struck out, and only the rest retained as possible subjects.

Selection. Suppose we now require an expression for the term *b* as inferred from the premise A=BC. Select from the included and excluded subjects such as contain *b*, namely *ab*C and *abc*.

Elimination. Then $b=abC+abc$, but as *a*C occurs only with *b*, and not with B, its contrary, we may, by Rule 6, eliminate *b* from *ab*C; hence $b=aC+abc$.

Elimination explained. **117.** The validity of this last elimination is seen by drawing the expression for *a*C, which is *ab*C. Then between $b=abC+abc$, and $abC=aC$, we may eliminate *ab*C by substituting (§ 51) its expression *a*C. And similarly in all other cases to which the rule applies.

We might also reduce the expression for *b* by Rule 5, as follows:—

$$b=abC+abc=ab\,(C+c)=ab.$$

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118. To express a we have *Other inference.*

$$a = aBc + abC + abc,$$

but observing that none of Bc, bC, bc, occur with A, so that $Bc = aBc$, $bC = abC$, $bc = abc$, we substitute these simpler terms, eliminating a; whence $a = Bc + bC + bc$, an evident truth (§ 113).

119. Similarly, we may draw any of the following inferences:— *Other inferences.*

$$A = ABC = AB = AC$$
$$B = AC + aBc$$
$$C = AB + abC$$
$$c = aB + abc = ac$$
$$aB = Bc$$
$$aC = bC$$
$$ab = abC + abc = ab \text{ (no inference)}$$
$$ac = aBc + abc = ac \text{ (no inference).}$$

120. Observe that since B and C are samely related to A, we may get any inference concerning one of these terms from the similar inference concerning the other by interchanging B and C, b and c (§ 56). *Relation of* B *and* C.

Before proceeding to further examples of indirect inference, we may make the following observations.

121. When any term appears on both sides of a premise, as A in $AB = AC$, any combination containing its contrary, a, is an excluded subject. Thus, in combining any term with both sides of a proposition, we render any contrary of the term an excluded subject. *Excluded subjects.*

So, in mathematics we introduce a new root into an equation when we multiply both sides by a factor.

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Of inferior importance.

122. An excluded subject, though admitting of inference and admitted into inferences, is of inferior and often of no importance. As its name expresses, it is usually a combination concerning which we do not desire knowledge. The sphere of an argument, or the *Universe of Thought*, contains all the included subjects. An excluded subject is such as lies beyond this sphere or universe. But we are obliged to consider excluded subjects, because the excluded subject of one premise may be the included subject of other premises.

Plural premises.

123. When a premise is plural in one or both sides, an excluded subject is a contrary of all the alternatives on both sides, and a contradictory combination is a contrary of all on one side, and not of all on the other side.

Identical proposition.

124. Of an identical proposition the term itself appearing on either side is the only included subject. All others are excluded, and there are no contradictory combinations. Its useless nature is thus evident.

Common subject.

125. Any subject of a proposition remains an included, excluded, or contradictory subject as before, after combination with any unrelated terms. Thus, if the argument be restricted to a sphere or *common subject*, defined by certain terms, these do not need expression in each premise, but may be retained as an exterior condition. Thus, by ABCD (. . . . . .) we might mean that ABCD is to be understood as combined with each term of any premises placed within the brackets. ABCD is then the common subject of the premises, which must contain no contrary of this. And any con-

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trary of ABCD is an excluded subject of the whole.

Of unbroken combinations.

126. Any set of terms which always occur in the premises in unbroken combination may be treated as a simple term.

Thus, if BC occur always thus in combination, we may write for it, say D, and then *d* or *not*-BC is $bC+Bc+bc$.

Unbroken plural terms.

127. Any set of alternatives which always occur together in the premises as alternatives may be treated as a single term.

Thus, if B and C occur always as alternatives, we may for B+C write, say D, and then *d* or *neither B nor C* is *bc*.

Simple proposition.

128. Any proposition may be treated under the form of A=B, so long as we do not require to treat its part-terms or alternatives separately. (By §§ 126, 127.)

Technical terms.

129. Hence the convenience in every branch of knowledge of using technical terms to stand for every large set of terms which usually occur together. But such terms become the source of error if we do not carefully keep before us their definitions, those adopted premises in which we express the set of combined or alternative terms for which we substitute a technical term.

Metaphysical terms.

130. In that branch of knowledge, however, called First Philosophy, which is *analytic*, and aims at resolving things, or our thoughts about them, into their simplest components, the use of technical terms is fallacious. Such terms cannot assist analysis, since each arises from the synthesis of many simpler terms, forming its definition.

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All reasoning, then, in Metaphysics or First Philosophy, ought to be carried on in the simplest and most vernacular elements of speech. Analytic science should be like a mill which grinds down the ordinary grains of thought into their smallest and simplest particles. It is in the bakehouse we should combine these particles again into loaves of a size and consistency suitable for ordinary use. But most metaphysical reasoners, it seems to me, have mistaken the mill and the bakehouse.

Interrupted process.

131. It is not always necessary to carry out the process of inference exactly as in the rules. Each or any premise may be treated as a separate one, if desirable, and its possible subjects afterwards combined with the possible subjects of other premises. We may thus successively add premises, or try the effect of supposed ones.

For instance, since AB and *ab* are the possible subjects of A=B, and BC and *bc* of B=C, the possible combinations of these, namely ABC and *abc*, are the possible subjects of the two premises combined, observing that AB*bc* and *ab*BC are contradictory.

Unrelated premises.

132. If premises be related, the indirect inferences will include all possible direct inferences. From unrelated premises we shall also get such inferences as are possible.

Thus, from the unrelated premises

$$A=B,\ C=D,$$

we have

$$A=BCD+Bcd$$
$$d=ABc+abc, \text{ and so on.}$$

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Proof of indirect method.

133. It does not seem possible to give any general proof that the conclusions of the indirect method must agree with those of the direct method, which will make its truth any the more evident. Such proof could be little less than a general recapitulation of the several Laws of Thought.

Euclid's indirect demonstration.

134. It hardly needs to be pointed out that the method of indirect inference is equivalent to Euclid's indirect demonstration, or *reductio ad absurdum*. Euclid assumes the developement of alternatives, usually that of *equal or greater or less*, and showing that two of these lead to a contradiction, establishes the truth of the third.

Common use of indirect method.

135. Nor is this process of reasoning at all new or uncommon in any branch of knowledge save logic, which was supposed to be the science of all reasoning. Simple instances occur perhaps as frequently as instances of direct inference, and complicated instances are only rendered scarce by the limited powers of human memory and attention. Among instances of indirect argument we may place all those discourses in which a writer or speaker states several possible alternatives or cases of his subject, and, after showing some of them to be impossible, concludes the rest to be necessary, or else proceeds further to develope and consider these with regard to other premises (§ 131). A good instance is found in Paley's Argument on the Divine Benevolence (Moral Phil., Book II. chap. V.). The old logical process called *abscissio infiniti* has a close relation to indirect inference.

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Quotation.

136. Even brute animals, it would seem, may reason by the indirect method:—

'This creature, saith Chrysippus (of the dog), is not void of Logick: for, when in following any beast he cometh to three several ways, he smelleth to the one, and then to the second; and if he find that the beast which he pursueth be not fled one of these two ways, he presently, without smelling any further to it, taketh the third way; which, saith the same Philosopher, is as if he reasoned thus: the Beast must be gone either this, or this, or the other way; but neither this nor this; *Ergo*, the third: so away he runneth.'

SIR W. RALEIGH.

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CHAPTER XII.

OF RELATION TO COMMON LOGIC.

Before giving examples of the processes of logical inference as now set forth, it will be well to consider the relation of our system to the logic of common thought.

Ordinary proposition imperfect.

137. In ordinary reasoning it will be found that there is great economy of thought. Not only are large collections of attributes and things grouped together under the fewest possible terms, but only those particular attributes of the things under consideration on which the reasoning turns are brought forward. A certain natural disinclination to exertion causes us to simplify our modes of thought as much as possible, and to leave in the background everything that is not essential. Thus when we say *man is mortal*, we mean that the attributes of mortality are among the attributes of man. But we leave out those infinitely numerous attributes of man which are not comprised under mortality, because we do not happen to be occupied with them. The proposition, then, in this form is not that equation of qualities, that statement of perfect sameness or equivalence of meaning, which we have taken as a proposition.

138. It may be objected that we ought to take

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Equation or sameness the true form of reasoning.

the proposition as we find it in common thought. Aristotle so took it, and his system has had a long reign. Some of the expounders of his system even denied that there could be a proposition of two universal and equivalent terms. They could not have committed a greater error or more completely misrepresented the ordinary course of reasoning. Not only, as a fact, do the several sciences establish multitudes of propositions of which the two terms are equivalent and universal, but all definitions are propositions of this kind, and the definitions requisite in connecting the meanings of more and less complex terms, must always form a large part of our data in reasoning. If we further consider that even Aristotle's negative propositions have a universal predicate, that men show a constant tendency to treat the predicate of the proposition A as universal, whence several common kinds of fallacy, and that *reasoning from same to same things may be detected as the fundamental principle of all the sciences*, we need have no hesitation in treating the equation as the true proposition, and Aristotle's form as an imperfect proposition.

It is thus the Law of Sameness, not the *dictum* of Aristotle, which governs reasoning.

Quantification of predicate.

139. It is only of very late years that the imperfection of the ordinary proposition has been properly pointed out. It is the discovery of the so-called *quantification of the predicate* which has reduced the proposition to the form of a convertible equation, and opened out to logic an indefinite field of improvement.

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140. Professor Boole's system, first published in his '*Mathematical Analysis of Logic*,' in 1847, involves this newly discovered quantification of the predicate. According to Boole, the *some*, which is the adjective of particular logical quality, is an *indefinite class symbol*. *Men are some mortals* is expressed by him in the equation, $x=vy$, where x instructs us to select from the universe all things that are men, and y to select all things that are mortal. The proposition then informs us that the things which are men consist of an indefinite selection from among the things which are mortal, v being the symbol of this indefinite quantity or class selected. *Boole's 'Analysis.'*

141. One more step seems to me necessary. It is to separate completely the qualitative and quantitative meanings of all logical terms, including the word *some*. In the qualitative form of the proposition *man is some mortal*—or more correctly speaking, *man is some kind of mortal*—we interpret *some* or *some kind* as meaning an indefinite and perhaps unknown collection of qualities, which being added to the qualities *mortal*, give the known qualities of *man*. In the quantitative form *men are some mortals*, we have the equivalent statement that the collection of individuals in the class *some mortals* is the collection of individuals in the class *men*. *Further step requisite.*

142. It is strange that the purely qualitative form of proposition *man is some kind of mortal*, which is the most distinct form of statement, and is perhaps the most prevalent, both in science and ordinary thought, was totally disregarded by *Qualitative proposition disregarded.*

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logicians, at least as the foundation of a system of logic. 'The Logicians, until our day,' says Professor De Morgan,* 'have considered the extent of a term as the only object of logic, under the name of the *logical whole*: the *intent* was called by them the *metaphysical whole*, and was excluded from logic.'

'Some' and 'some kind.'

143. It will be seen that this word *some* or *some kind*, the source of so much difficulty and error, must in our system be treated as a term of indefinite and unknown meaning. It is an unknown term, not only at the beginning of a problem, but throughout it. In no two premises then can the term *some* or *some kind* be taken to mean the same set of qualities. Thus we cannot argue through or eliminate a term with *some*, while at least it retains this unknown term: that is to say, we can never use it as a common term (§ 27) in direct inference. Thus, if A is *some* B, and *some* B is *some* C, we cannot eliminate *some* B getting A *is some* C, because *some* being of unknown meaning, the *some* B is not necessarily the same in both cases. This is still more plain in the form *A is some kind of B*, and *some kind of B is some kind of C*, for it is obvious that the one kind of B is not necessarily the same as the other.

Symbol for 'some.'

144. Since the term *some* or *some kind* is not only unknown but remains unknown throughout any argument, we might conveniently appropriate to it some symbol such as U, to remind us of its special conditions. Thus no term U is to be taken

* *Syllabus*, p. 61.

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as same with any other term U, or U=U is not known to be true. But in the propositions A and E it is always open to us, and is best to eliminate U by writing for it the other member of the proposition (§ 52). Thus, A=UB, meaning that A is *some kind* of B, involves three terms. It is much better written as A=AB, involving only A and B, and yet perfectly expressing that the qualities of B are among those of A, but not necessarily those of A all among those of B.

Aristotle's propositions.

145. The four propositions of the old logic may thus find expression in our system :—

A .	*Every A is B*	A=UB	or	A=AB
E .	*No A is B*	A=U*b*	or	A=A*b*
I .	*Some A is B*	UA=UB	or	CA=DB
O .	*Some A is not B*	UA=U*b*	or	CA=D*b*.

De Morgan's propositions.

146. Two new propositions of De Morgan's system are thus expressed :—

Everything is either A or B	A=*b*
Some things are neither A nor B	*a* =*b*.

Thomson's proposition.

147. All these propositions, and as many more as may be proposed, can be brought and partially treated (§ 128) under the form A=B, which I believe to be the simple form of all reasoning. The existence of doubly-universal propositions of this kind was far from being unknown to many of the School Logicians, but out of deference to the Aristotelian system, such propositions were neglected. The present Archbishop of York first embodied this proposition in a system of logic, giving it the name U. (Thomson's 'Outlines,' *passim.*)

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CHAPTER XIII.

EXAMPLES OF THE METHOD.

In this chapter I shall place some miscellaneous examples of inference according to the system of the foregoing chapters, suited to show the power of its method, or its relation to the old logic.

Syllogism in Felapton.

148. Let us take a syllogism in FELAPTON.

No A is B	$A=Ub=Ab$
Every A is C	$A=UC=AC$
Some C is not B	(§ 145.)

Direct inference.

From $A=Ab$ we might by combination (§ 45) infer $AC=AbC$, and from $A=AC$, $Ab=AbC$; whence $AC=AbC=Ab$, or $AC=Ab$, *which is a more precise statement of* $UC=Ub$, or *some* C *is not* B, the Aristotelian conclusion.

We may, however, obtain this conclusion, as well as all other possible ones, by indirect inference.

AbC
aBC
aBc
abC
abc

Of the possible combinations of A, B, C, *a*, *b*, *c*, ABC and ABc are contradicted by the first premise, and A*bc* (as well as ABc) is contradicted by the second premise. AbC, aBC, aBc, abC, abc, are the remaining combinations in which we find there is no relation between B and C *per se*, since B occurs with C and *c*, and C occurs with B and *b*. But

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$AC = AbC$, and $Ab = AbC$, whence, by elimination, $AC = Ab$, the same conclusion as before.

The following conclusions may also be drawn:

$$a = a(BC + Bc + bC + bc) = a(B + b)(C + c)$$
$$= a \text{ (no inference)}$$

$$B = aB$$
$$b = AC + abC + abc = AC + ab$$
$$C = Ab + aBC + abC = Ab + aC$$
$$c = aBc + abc = ac$$
$$ab = abC + abc = ab(C + c) = ab \text{ (no inference)}$$
$$aC = aBC + abC = aC(B + b) = aC \text{ (no inference)}$$
$$bc = abc.$$

149. The premise $AB = CD$ is of some interest. It contradicts the combinations $ABCd$, $ABcD$, $ABcd$, which are AB and not CD, and $AbCD$, $aBCD$, $abCD$, which are CD and not AB. From the remainder we easily draw the inferences

Example: $AB = CD$

$ABCD$
$AbCd$
$AbcD$
$Abcd$
$aBCd$
$aBcD$
$aBcd$
$abCd$
$abcD$
$abcd$

$$A = BCD + AbCd + AbcD + Abcd$$
$$a = BCd + BcD + Bcd + abCd + abcD + abcd.$$

Observing that A and B enter samely into the premise, we may easily deduce the expressions for B and b by interchanging A and B, a and b in the above; thus (§§ 54-56)—

$$B = ACD + aBCd + aBcD + aBcd.$$

And since A, B enter samely with C, D, we might deduce the corresponding expressions for C, D, and c, a, by interchanging at once A with C, and B with D, or A with D, and B with C.

From the expression for a we thus get

$$d = CBa + CbA + Cba + dcBa + dcbA + dcba.$$

Observe, that if the expression for A be com-

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bined with that for a, nothing but contradictory terms will be the result, verifying $Aa=0$. And, if we combine the expressions for any terms not contrary, as B and d, we get the same result as we might have drawn by the separate application of the process.

Thus, $Bd=AbCD+aBCd+aBcd=0+aBd$.

In expressions thus derived there will often appear, as in the above instance, a superfluous and contradictory term ($AbCD$, a contrary of Bd), but being only an alternative, the proposition is not untrue.

Example: $A=B+C$

150. As an example of a premise with a plural term, let us take $A=B+C$.

ABC
ABc
AbC
abc

In comparing the eight combinations of A, B, C, a, b, c, with the premise, any one is contradictory which contains A without containing either B or C; or, again, which contains either B or C without containing A. Thus, ABC, ABc, and AbC, are the included subjects, abc is an excluded subject, and the rest are contradictory.

We may draw the inferences

$$\begin{aligned} A&=BC+Bc+bC \\ a&=bc \\ B&=ABC+Ac=AB \\ b&=AbC+a \\ C&=ABC+Ab=AC \\ c&=ABc+a. \end{aligned}$$

Observe that B and C enter samely, so that their expressions may be mutually derived by interchange.

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151. The premise $A=Bc+bC$ differs from the last in the very important point that A cannot at once be B and C.

Example: $A=Bc+bC$

It has the included subjects ABc and A*b*C, and the excluded subjects *a*BC and *abc*. The following expressions are seen to be simple and symmetrical, and it is instructive to form their combinations.

ABc
A*b*C
*a*BC
abc

$$A=Bc+bC$$
$$a=BC+bc$$
$$B=Ac+aC$$
$$b=AC+ac$$
$$C=Ab+aB$$
$$c=AB+ab.$$

152. From two negative premises we can infer no Aristotelian conclusion (§§ 77, 78). It is well to show that this remains true when the negative propositions are converted into their corresponding affirmatives (§ 107).

Negative premises.

Let us take the premises

A is not the same as B,
C is not the same as B.

These may be expressed by the affirmative propositions $A=Ab$, $C=bC$.

If we go through the process of indirect inference, and attempt to express A and C in terms of each other, we shall obtain :—

A*b*C
A*bc*
*a*B*c*
*ab*C
abc

$$A=AbC+Abc=Ab\ (C+c)=Ab$$
$$C=AbC+abC=bC\ \ (A+a)=bC.$$

These are the premises over again, and there can be no new inference, except $B=aBc$.

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Solutions of form A = B

153. The proposition A = B being the simplest form of statement, its full solution is given below, and the solutions of the similar propositions A = *b*, *a* = B, *a* = *b*, are inferred by interchanging A and *a*, B and *b*.

Premise	A = B	A = *b*	*a* = B	*a* = *b*
Included subject	AB	A*b*	*a*B	*ab*
Excluded subject	*ab*	*a*B	A*b*	AB
Contradiction	A*b*	AB	AB	*a*B
Contradiction	*a*B	*ab*	*ab*	A*b*.

Example of two premises.

154. Let us take A = ABC,
B + C = BD + CD.

We have, by direct inference from the second premise, BC = BCD (§ 45)
Hence A = ABC = ABCD (§ 26.)

ABCD
*a*BCD
*a*B*c*D
*ab*CD
*abc*D
abcd

The indirect process gives four included and two excluded subjects, as in the margin.

Hence not only the above inference, but the following, among other possible ones:—

$$a = a\text{BCD} + a\text{B}c\text{D} + ab\text{CD} + abc\text{D} + abcd$$
$$= a\text{BD} + ab\text{D} + abd$$
$$= a\text{D} + abd = a\text{BD} + ab$$
$$\text{C} = \text{ABD} + a\text{BCD} + ab\text{CD} = \text{ABD} + a\text{CD}$$
$$c = a\text{B}c\text{D} + abc\text{D} + abcd = ac\text{D} + abd$$
$$\text{B}c = a\text{B}c\text{D} \qquad c\text{D} = ac\text{D}$$

Sorites.

155. The ordinary Sorites is easily and clearly solved in this system. Taking four premises such as A = AB
B = BC
C = CD
D = DE, many inferences will be evident

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from the following series of the subjects, or possible combinations.

ABCDE *a*BCDE *ab*CDE *abc*DE	Included subjects.
*abcd*E *abcde*	Excluded subjects.

156. The Dilemma of the old logic is easily included in our system, when we supply a term which is suppressed or understood in its usual statement. The dilemma is as follows:— *Dilemma.*

If A is B, E is F, and if C is D, E is F: but, either A is B, or C is D, therefore E is F. Adopting Wallis's reduction to the categorical form, and supplying some term G, to express the *present circumstances*, or the *case* in which either A is B, or C is D, we have the premises

$$AB = ABEF$$
$$CD = CDEF$$
$$G = ABG + CDG.$$

By the direct process alone we get the required conclusion that, under the condition G, E is F; thus—

$$GE = (AB + CD)\,GE = ABEFG + CDEFG = GEF$$
$$\text{or, } GE = GEF.$$

157. The following is known as a Destructive Conditional Syllogism. *Destructive conditional syllogism.*

If A is B, C is D; but C is not D; therefore, A is not B.

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Supplying the suppressed term, say E, expressing the circumstances in which A is not B, the following is the statement of this syllogism in our system :—

$$AB = ABCD$$
$$CE = Cd E.$$

By direct inference

$$ABE = ABD.CE = ABD.Cd E = 0.$$

Hence ABE is known to be contradictory; therefore (§ 108), AE is not ABE, or in the circumstances E, A is not B.

Forms of old logic.

158. The forms of the old logic being comprehended in this system along with an indefinite multitude of other forms, logicians can only properly accept this generalisation, due to Boole, by throwing off as dead encumbrances the useless distinctions of the Aristotelian system. The past history of the Science must not, as hitherto, bar its progress. And Logic will be developed almost like Mathematics, when Logicians like Mathematicians discriminate between the Study of Thought and the Study of Antiquarian Lore.

I will now give a few complex problems, more suited to show the power of the method.

Complex problem.

159. Let the premises be

$$A = B + C$$
$$B = c + d$$
$$c = cD$$
$$AD = BCD.$$

And let it be required to infer the description of any term, say a. By the indirect process, we shall find that the only combination uncontradicted by one or other premise is $ABCd$.

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Thus, we find there cannot be any a at all, without contradiction, whatever may be the meaning of this result.* It means, doubtless, that the premises are contradictory. (§ 115.7.)

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* The following law, being of a less evident character than the rest, has been placed apart. *Law of infinity.*

Every logical term must have its contrary.

That is to say:—*Whatever quality we treat as present we may also treat as absent.*

There is thus no boundary to the universe of logic. No term can be proposed wide enough to cover its whole sphere; for the contrary of any term must add a sphere of indefinite magnitude. Let U be the universe; then u is not included in U. Nor will special terms limit the universe. *Universe of logic unbounded.*

Thing existing has its contrary in *thing not existing.*

Thing thinkable has its contrary in *thing not thinkable.*

Even *thing*, the widest noun in the language, has a contrary in *that which is not a thing.*

Of course the above is only true speaking in the strictest logical sense, and using all terms in the most perfect generality.

If the above be granted as true, every proposition of the form $A = B + b$ must be regarded as contradictory of a law of thought. For the contrary of A from the above is Bb, a contradiction, or A is used as having no contrary, and forming the universe. *Contradictory proposition.*

Also every system of premises must be rejected which altogether contradicts any term or terms. Thus in the indirect process we must always have at least two combinations remaining possible, one of which must contain the contrary of each simple term in the other. In this view the peculiar premises *Contradictory premises.*

$$A = B + C$$
$$B = c + d$$

(§ 159) contain subtle contradictions. For a according to the first premise must be bc, and being c, it must by the second premise be B, and hence by first premise also, A, or both A and a, B and b.

But this subject needs more consideration.

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We also easily infer any of the following:—

$$\begin{array}{lll} A=BCd & AB=Cd & ABC=ABCd \\ B=ACd & AC=Bd & ABd=ABCd \\ C=ABd & BC=Ad & \text{etc.} \\ d=ABC. & & \end{array}$$

Problem.

160. The following premises are such as might easily occur in physical science:—

$$\begin{aligned} A&=ABc+Ab\mathrm{C} \\ B&=BDE+Bde \\ C&=CDe. \end{aligned}$$

ABcDE
ABcde
AbCDe
aBcDE
aBcde
abCDe
abcDE
abcDe
abcdE
abcde

The series of possible combinations in the margin gives by inspection perhaps the most useful information, but the following are a few formal inferences.

$$\begin{aligned} A&=ABcDE+ABcde+AbCDe \\ BcD&=BcDE \\ abd&=abcd \\ cd&=ABcde+aBcde+abcd(E+e) \\ &=Bcde+abd \\ bCD&=AbCDe+abCDe \\ &=bCDe. \end{aligned}$$

There is no relation between *abc* and D and E.

Complicated problem.

161. I conclude with the solution of a still more complicated system of premises.

$$\begin{aligned} A+C+E&=B+D+F \\ Bc+bC&=De+dE \\ AD&=ADf \\ D&=e \\ C&=Cd. \end{aligned}$$

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The possible combinations are:—

ABcD*ef*	*a*Bc*d*EF
ABc*d*EF	*a*Bc*d*E*f*
ABc*d*E*f*	*ab*C*d*EF
A*b*C*d*EF.	

Whence the following, among many other inferences, may be drawn:—

$$\begin{aligned}
A &= BcDef + ABcdE + AbCdEF \\
Bc &= ADef + BcdE \\
D &= ABcef \\
cd &= BE \\
e &= ABcDf \\
dE &= ABcd + bCF + aBc = Bcd + bCF \\
bdE &= bCF = CdE \\
AF &= ABcdEF + AbCdEF \\
aF &= aBcdEF + abCdEF \\
df &= BcdEf \\
b &= bCdEF \\
C &= bCdEF = b.
\end{aligned}$$

Whence the remarkable and unexpected relation $C=b$, which it would not be easy to detect in the premises.

Verification.

162. Inferences may be verified by combining the expressions of two or more terms, and comparing the result with the expression of the combined term as drawn from the series of possible combinations. For instance, in the problem last given (§ 161), we may combine the expression for A with that for dE, as follows:—

$$\begin{aligned}
A.dE &= (BcDef + ABcdE + AbCdEF)(Bcd + bCF) \\
&= 0 + ABcdE + 0 + 0 + AbCdEF,
\end{aligned}$$

the contradictory combinations being struck out.

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But the expressions thus obtained may not always be in their simplest terms.

Reduction not necessary.

163. The reduction of inferences to their simplest terms, it may be remarked, is in no way essential to their truth; it only renders them more pregnant with information. It is, perhaps, the only part of the process in which there is any difficulty.

Working of the process.

164. In working these logical problems, it has been found very convenient to have a series of combinations of terms beginning with those of A, B, and proceeding up to those of A, B, C, D, E, F, or more, *engraved* upon a common writing slate. In any given problem, the series is chosen which just furnishes sufficient letters for the distinct terms. The contradictory combinations may then be rapidly struck out, and the remaining combinations lie ready before the eye.

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CHAPTER XIV.

COMPARISON WITH BOOLE'S SYSTEM.

165. To show the power and facility of this method, as compared with that of Professor Boole, it will be sufficient, as regards those already acquainted with Professor Boole's system, to present the solution of one of his complex examples. Thus, let us follow Professor Boole's investigation of Senior's definition of wealth, namely*—that *wealth is what is transferable, limited in supply, and either productive of pleasure or preventive of pain.* (Boole, p. 106.)

Senior's definition of wealth.

Let A = Wealth
B = Transferable
C = Limited in supply
D = Productive of pleasure
E = Preventive of pain.

The definition in question is expressed by the proposition

$$A = BC(DE + De + dE)$$

which includes all the combinations of D, E, *d*, *e*, except *de*.

* Here, as usually elsewhere, I take words in intent of meaning, and transform most statements accordingly.

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Striking out the dual term $(E+e)$ from $BCD(E+e)$, we may state the definition in the more concise form

$$A=BCD+BCdE.$$

We may pass over Prof. Boole's expression for A, after intrinsic elimination of E ($A=BCD+ABCd$), as being sufficiently obvious.

Expression for C.

166. *Required* C *in terms of* A, B, D (Boole, p. 107).

ABCDE
ABCD*e*
ABC*d*E
...
...
*a*BC*de*
*a*B*c*DE
*a*B*c*D*e*
*a*B*cd*E
*a*B*cde*
*ab*CDE
*ab*CD*e*
*ab*C*d*E
*ab*C*de*
*abc*DE
*abc*D*e*
*abcd*E
abcde

Forming all the possible combinations of A, B, C, D, E, and their contraries, and comparing them with the premise, we shall find all the combinations from ABC*de* to *a*BC*d*E inclusive contradicted. The remaining subjects are as in the margin.

Selecting the terms containing C, we have

$$C=ABCDE+ABCDe+ABCdE+aBCde$$
$$+abCDE\ +abCDe\ +abCdE\ +abCde.$$

Striking out the dual terms $(E+e)$, and intrinsically eliminating remaining E's or *e*'s by substitution of C, we have

$$C=ABCD+ABCd+aBCd+abCD+abCd.$$

Eliminating C from ABCD (§ 117), because ABD=ABCD, and striking out the dual terms $(A+a)$ and $(D+d)$, we have either of the expressions—

$$C=ABD+BCd\ +abC$$
$$C=ABC+aBCd+abC.$$

From the latter we read, *What is limited in supply is either wealth, transferable* (*and either productive of pleasure or not*, ABC), *or else some*

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kind of what is not wealth, but is either not transferable (*ab*C), *or, if transferable, is not productive of pleasure* (*a*BC*d*).

This conclusion is exactly equivalent to that of Professor Boole, on p. 108.

Negative conclusions.

167. His so-called secondary propositions, namely, '1. Wealth that is intransferable and productive of pleasure, does not exist;' and '2. Wealth that is intransferable and not productive of pleasure does not exist,' are negative conclusions implied in the striking out of the contradictory combinations A*b*CDE, A*b*CD*e*, A*bc*DE, A*bc*D*e*, and A*b*C*d*E, A*b*C*de*, A*bcd*E, A*bcde*, which are easily reducible to

$$Ab\mathrm{D}\ (\mathrm{C}+c)\ (\mathrm{E}+e)=0 \qquad Ab\mathrm{D}=0$$
$$Abd\ (\mathrm{C}+c)\ (\mathrm{E}+e)=0 \qquad Abd=0.$$

The expression 'does not exist' is open to exception.

Expression for D.

168. Again, required an expression for productive of pleasure (D), in terms of wealth (A), and preventive of pain (E). (Boole, p. 111.)

The complete collection of combinations containing D is

ABCDE	*ab*CDE
ABCD*e*	*ab*CD*e*
*a*B*c*DE	*abc*DE
*a*B*c*D*e*	*abc*D*e*.

We may then write D as follows:—

$$\mathrm{D}=\mathrm{ABCDE}+\mathrm{ABCD}e+a(\mathrm{B}c+b\mathrm{C}+bc)(\mathrm{E}+e)\mathrm{D}.$$

But we may observe also that

$$\mathrm{ADE}=\mathrm{ABCDE} \quad \text{and} \quad Ae=\mathrm{ABCD}e.$$

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Hence we may substitute ADE and Ae for the two first terms of the expression for D. We may also strike out the dual term (E+e) in the third term, and eliminate the plural term (Bc+bC+bc) intrinsically (§ 52) by substitution of D. Thus we get the expression in the required terms:

$$D=ADE+Ae+aD,$$

which may be translated into these words:—*What is productive of happiness is either some kind of wealth preventive of pain, or any kind of wealth not preventive of pain, or some kind of what is not wealth.* (Boole, p. 111 *ad fin.*)

Expression for d.

169. For the expression of d we easily select

$$d=Ad\mathrm{E}+ade+ad\mathrm{E}=A\mathrm{E}+ad$$

of which the meaning is—*What is not productive of pleasure is either some kind of wealth preventive of pain, or some kind of what is not wealth.* (Boole, p. 112.)

Other inferences.

170. These are the chief inferences furnished by Mr. Boole. From the list of possible combinations we could easily add a great many more inferences, in fact as many more, as may be drawn concerning any of the five terms A, B, C, D, E, and their contraries.

Thus for CE expressed in the remaining terms, we have

$$\begin{aligned}\mathrm{CE}&=\mathrm{ABCDE}+\mathrm{ABC}d\mathrm{E}+ab\mathrm{CDE}+ab\mathrm{C}d\mathrm{E}\\&=(\mathrm{ABCE}+ab\mathrm{CE})\,(\mathrm{D}+d).\end{aligned}$$

Striking out the dual term (D+d) and extrinsically eliminating C in ABCE, since we observe that ABE=ABCE, we have

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$$CE = ABE + abCE$$

which may be translated—

What is limited in supply, and preventive of pain, is either wealth, transferable and preventive of pain, or some kind of what is not wealth and not transferable.

But we may often find that there is no special relation to express. Thus, in trying to express $abCD$ in terms of E we find

$$abCD = abCD\ (E + e) = abCD.$$

171. Besides affording these formal deductions, the series of possible combinations will often give us at a glance a clear and valuable notion of the manner in which the universe of our subject is made up. *General character of combinations.*

In this instance we see that for wealth we have the three combinations BCDE, BCDe and BCdE, and that thus for *not-wealth* (a) we have all possible combinations of B, C, D, E, except those three. With aB we have Cde and c (DE + De + dE + de), and with ab, we have all possible combinations of C, D and E. Thus the definition gives no relation between what is not wealth and not transferable, and what is limited in supply, productive of pleasure, or preventive of pain.

172. It is the character of this logical system, in common with that of Professor Boole, that it is perfectly general. The same rules which govern the inferences from one or two premises, involving two or three terms, are applicable without the slightest modification to any number of premises, *Generality of the system.*

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involving any number of terms. Of course the working of the inferences becomes rapidly more laborious as the complexity of the problem increases, and a considerable liability to mistake arises. But this is in the nature of things, and the process of inference, consisting in the mere comparison of terms as to their sameness or difference, seems to me the simplest process that can be conceived.

Comparison with Boole's system.

173. Compared with Professor Boole's system, in its mathematical dress, this system shows the following advantages.

1. Every process is of self-evident nature and force, and governed by laws as simple and primary as those of Euclid's axioms.

2. The process is infallible, and gives no uninterpretable or anomalous results.

3. The inferences may be drawn with far less labour than in Professor Boole's system, which generally requires a separate computation and developement for each inference.

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CHAPTER XV.

REMARKS ON BOOLE'S SYSTEM, AND ON THE RELATION OF LOGIC AND MATHEMATICS.

174. So long as Professor Boole's system of mathematical logic was capable of giving results beyond the power of any other system, it had in this fact an impregnable stronghold. Those who were not prepared to draw the same inferences in some other manner could not quarrel with the manner of Professor Boole. But if it be true that the system of the foregoing chapters is of equal power with Professor Boole's system, the case is altered. There are now two systems of notation, giving the same formal results, one of which gives them with self-evident force and meaning, the other by dark and symbolic processes. The burden of proof is shifted, and it must be for the author or supporters of the dark system to show that it is in some way superior to the evident system.

175. It is not to be denied that Boole's system is consistent and perfect within itself. It is, perhaps, one of the most marvellous and admirable pieces of reasoning ever put together. Indeed, if Professor Ferrier, in his *Institutes of Metaphysics*, is right in holding that the chief excellence of a system is in being *reasoned* and consistent within itself, then Professor Boole's is nearly or quite the most perfect system ever struck out by a single writer.

176. But a system perfect within itself may not be a perfect representation of the natural system of human thought. The laws and conditions of thought as laid down in the system may not correspond to the laws and conditions of thought in reality. If so, the system will not be

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one of Pure and Natural Logic. Such is, I believe, the case. Professor Boole's system is Pure Logic fettered with a condition which converts it from a purely logical into a numerical system. His inferences are not logical inferences; hence they require to be *interpreted*, or translated back into logical inferences, which might have been had without ever quitting the self-evident processes of pure logic.

Among various objections which I might urge to Boole's system, *regarded as purely logical in purpose*, are four chief ones to which I shall here confine my attention.

First Objection.

177. *Boole's symbols are essentially different from the names or symbols of common discourse—his logic is not the logic of common thought.*

Professor Boole uses the symbol + to join terms together, on the understanding that they are logical contraries, which cannot be predicated of the same thing or combined together without contradiction. He says (p. 32)—'In strictness, the words "and," "or," interposed between the terms descriptive of two or more classes of objects, imply that those classes are quite distinct, so that no member of one is found in another.'

178. This I altogether dispute. In the ordinary use of these conjunctions, we do not necessarily join logical contraries only; and when terms so joined do prove to be logically contrary, it is by virtue of a *tacit premise*, something in the meaning of the names and our knowledge of them, which teaches us they are contrary. And when our knowledge of the meanings of the words joined is defective, it will often be impossible to decide whether terms joined by conjunctions are contrary or not.

179. Take, for instance, the proposition—'A peer is either a duke, or a marquis, or an earl, or a viscount, or a baron.' If expressed in Professor Boole's symbols, it would be implied that a peer cannot be at once a duke and marquis, or marquis and earl. Yet many peers do possess two or more titles, and the Prince of Wales is Duke of Cornwall, Earl of Chester, Baron Renfrew, &c. If it were enacted

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by parliament that no peer should have more than one title, this would be the tacit premise which Professor Boole assumes to exist.

Again,—'Academic graduates are either bachelors, masters, or doctors' does not imply that a graduate can be only one of these; the higher degree does not annul the lower.

Shakespeare's lines,—

> 'Beauty, truth, and rarity,
> Grace in all simplicity,
> Here inclosed in cinders lie.
> * * * *
> To this urn let those repair
> That are either true or fair,'—

certainly do not imply that beauty, truth, rarity, grace, and the true and fair are incompatible notions, so that no instance of one is an instance of another.

In the sentence—'Repentance is not a single act, but a habit or virtue,' it cannot be implied that a virtue is not a habit; by Aristotle's definition it is.

Milton has the expression in one of his sonnets—'Unstain'd by gold or fee,' where it is obvious that if the fee is not always gold, the gold is a fee or bribe.

Tennyson has the expression 'wreath or anadem.' Most readers would be quite uncertain whether a wreath may be an anadem, or an anadem a wreath, or whether they are quite distinct or quite the same.

From Darwin's 'Origin,' I take the expression, 'When we see any *part or organ* developed in a remarkable *degree or manner*.' In this, *or* is used twice, and neither time disjunctively. For if *part* and *organ* are not synonymous, at any rate an organ is a part. And it is obvious that a part may be developed at the same time both in an extraordinary degree and manner, although such cases may be comparatively rare.

180. From a careful examination of ordinary writings, it will be found that the meanings of terms joined by 'and' 'or' vary from absolute identity up to absolute contrariety. There is no logical condition of contrariety at all, and when we do choose contrary expressions, it is because our

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subject demands it. The matter, not the form of an expression, points out whether terms are exclusive. And if there is one point on which logicians are agreed, it is that logic is formal, and pays no regard to anything not formally expressed. (See § 48.)

181. And if a further proof were wanted that Professor Boole's symbols do not correspond to those of language, we have only to turn to his own work. He actually translates one same sentence into different sets of symbols, according to the view he takes of the matter in hand. For instance, (p. 59) he interprets 'Either productive of pleasure or preventive of pain' so as not to exclude things both productive of pleasure and preventive of pain. 'It is plain,' he remarks, 'from the nature of the subject, that the expression "either productive of pleasure or preventive of pain," in the above definition, is meant to be equivalent to "either productive of pleasure; or if not productive of pleasure, preventive of pain."'

And in remarking upon other possible interpretations, he says, 'that before attempting to translate our data into the rigorous language of symbols, it is above all things necessary to ascertain the *intended* import of the words we are using.' (p. 60). This simply amounts to consulting the matter, and Professor Boole's symbols thus constantly imply restrictions not expressed in the forms of language, but existing, if at all, as tacit or understood premises.

182. In my system, on the contrary, I take A+B not to imply at all that A may not be B, but if this be the case, it must be owing to an expressed premise $A=Ab$ or $A=b$.

183. How essential Professor Boole's restriction on his symbols is to the stability of his system, one instance will show. Take his proposition on p. 35—

$$x=y+z,$$

and give the following meanings to x, y, z:—

x = Cæsar
y = Conqueror of the Gauls
z = First emperor of Rome.

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Now, there is nothing logically absurd in saying 'Cæsar is the conqueror of the Gauls or the first emperor of Rome.'

It is quite conceivable that a person should remember just enough of history to make this statement and nothing more. And there is nothing in the logical character of the terms to decide whether the conqueror could or could not be the same person as the first emperor.

But now take Professor Boole's inference from the proposition $x=y+z$, namely $x-z=y$ got by subtracting z from either side of $x=y+z$. Then we have the strange inference:—

Cæsar, provided he is not the first emperor of Rome, is the conqueror of the Gauls.

This leads me to my second objection to Professor Boole's system.

Second Objection.

184. *There are no such operations as addition and subtraction in pure logic.*

The operations of logic are the combination and separation of terms, or their meanings, corresponding to multiplication and division in mathematics. I cannot support this statement without going at once to the gist of the whole matter.

185. Number, then, and the science of number, arise out of logic, and the conditions of number are defined by logic. It has been thought that units are units inasmuch as they are perfectly similar. For instance, three apples are three units, inasmuch as each has exactly the same qualities as the other in being an apple. The truth is exactly opposite to this. *Units are units inasmuch as they are logically contrary.* In so far as three apples are exactly like each other, one could not be distinguished from the other. Were there three apples, or any three things, so perfectly similar in every way that we could not tell the difference, they would be but one thing, just as, by the law of unity before stated, $A+A+A=A$. But then we must remember that among the logical characters of a thing is its position in space with relation to other things, not to speak of its position in time. Now, when we speak of

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three apples, we mean three things, which, however perfectly same they may be in all other qualities, occupy different places, and are therefore distinct things. *In so far as they are same they are one; in that they are different they are three.*

186. The meaning of an abstract unit is *something only known as logically distinct from or contrary to other things.* The meaning of a concrete unit is *the abstract unit with certain qualities known or defined.*

For instance, in A $(1'+1''+1''')=A'+A''+A'''$ the meaning of the units $1'$, $1''$, $1'''$, is that each is something logically distinct from the other, and when we predicate of each of these that it is A, say an apple, we get three distinct A's, $A'+A''+A'''$.

So in multiplication, *twice two is four*—

$$(1+1)\,(1+1)=1+1+1+1.$$

The logical significance of the process is that if we have two logically distinct notions, and we divide each into two logically distinct notions, we get four logically distinct notions. In logical formulæ $(A+a)\,(B+b)=AB+Ab+aB+ab$, where A and a, B and b, express logical contraries.

187. Now addition, subtraction, multiplication, and division, are alike true as modes of reasoning in numbers, where we have the logical condition of a unit as a constant restriction. But addition and subtraction do not exist, and do not give true results, in a system of pure logic, free from the condition of number.

For instance, take the logical proposition—

$$A+B+C=A+D+E$$

Meaning *what is either A or B or C is either A or D or E*, and *vice versâ*.

There being no exterior restrictions of meaning whatever, except that the same term must always have the same meaning (§ 14), we do not know which of A, D, E, is B, nor which is C; nor, conversely, do we know which of A, B, C, is D, nor which is E. The proposition alone gives us no such information.

In these circumstances, the action of subtraction does not apply. It is not necessarily true that, if from same (equal)

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things we take same (equal) things the remainders are same (equal). It is not allowable for us to subtract the same thing (A) from both sides of the above proposition, and thence infer—

$$B+C=D+E.$$

This is not true if, for instance, each of B and C is the same as E, and D is the same as A, which has been taken away.

Yet the equivalent inference by combination will be valid. We may combine a with both sides of the proposition, and we have

$$aA+aB+aC=aA+aD+aE$$

or, striking out the contradictory terms aA, we have

$$aB+aC=aD+aE.$$

188. But subtraction is valid under the logical restriction that the several alternatives of a term shall be mutually exclusive or contrary. Let

$$(1) \qquad AMN+BM\eta+C\eta N=AMN+DM\eta+E\eta N$$

in which it is obviously impossible that AMN can be either $DM\eta$ or $E\eta N$, contraries of AMN, or any one of the three alternatives any other. Then we may freely subtract AMN from both sides, getting the necessary inference

$$(2) \qquad BM\eta+C\eta N=DM\eta+E\eta N.$$

This subtraction, however, is merely equivalent to the combination with both sides of the proposition (1) of the term $(M\eta+\eta N)$; for the combination being performed, and contradictory terms struck out, it will be found that the proposition (2) results.

189. In short, when alternatives are contraries of each other, subtraction of one is exactly equivalent to combination with the rest. The axiom (Boole, p. 36), that 'if equal things are taken from equal things, the remainders are equal,' is nothing but a case of the Law of Combination (§ 44), that if same (equal) terms be combined with same (equal) terms, the wholes are same (equal).

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Take the self-evident proposition

$$AB+Ab+aB+ab=AB+Ab+aB+ab$$

Any terms, say $aB+ab$, may be subtracted from both sides by combining the other terms $AB+Ab$ with each side of the proposition. Then

$$(AB+Ab)\ (AB+Ab+aB+ab)$$
$$=(AB+Ab)(AB+Ab+aB+ab)$$
$$AB+Ab+0\ \ .\ .\ +0=AB+Ab+0+\ .\ .\ 0+0.$$

And what is true of this self-evident case must be true when the premise is not self-evident.

190. Having thus established our liberty to subtract same terms, provided all alternatives are contraries, we have the corresponding liberty to add by the inverse process.

191. The processes of addition and subtraction thus arise out of the logical process of combination. The axioms of addition and subtraction are only valid under a logical condition, which is certainly not applicable to thought or language generally. And this condition is that which logic imposes upon number, that each two units shall be contrary logical alternatives. It is logic which reduces to a unit, by the Law of Unity, $A+A=A$, any two alternatives known to be the same, so that the science of number treating of units, treats of alternatives known to be different or contrary. But *logic itself is the superior science, and may treat of alternatives of which it is not known whether they are same or different.*

192. It is the self-evident logical Law of Unity, then, which lays the foundations of number. This law merely amounts to saying that a thing cannot and must not be distinguished from itself. We commit an error against this law, when in counting over coins, for instance, to ascertain their numbers, that is, how many logically distinct coins there are, we count the self-same coin two or more times, making the coins for instance

$$C'+C''+C''+C'''+C''''+\ .\ .\ .\ .\ .\ .$$

instead of $C'+C''+C'''+C''''+\ .\ .\ .\ .\ .$ It is by the Law of Unity that $C''+C''=C''$, or the same coin counted

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twice is but one coin in number. In this case no attention is paid to differences of time; but in many cases, things otherwise perfectly the same, like the beats of a pendulum, are distinguished and made into different units by one being before or after the other in time.

Third Objection.

193. My third objection to Professor Boole's system is, that *it is inconsistent with the self-evident law of thought, the Law of Unity.* (A+A=A.)

Prof. Boole having assumed as a condition of his system that each two terms must be logically distinct, is unable to recognise the Law of Unity. It is contradictory of the basis of his system. The term x, in his system, means *all things with the quality* x, denoting the things in extent, while connoting the quality in intent. If by 1 we denote all things of every quality, and then subtract, as in numbers, all those things which have the quality x, the remainder must consist of all things of the quality *not-x*. Thus, $x+(1-x)$ means in his system *all x's with all not-x's*, which, taken together, must make up all things, or 1. But let us now attempt by multiplication with x, to select *all x's* from this expression for *all things*.

$$x(x+1-x)=x+x-x.$$

Professor Boole would here cross out one $+x$ against one $-x$, leaving one $+x$, the required expression for *all x's*. It is surely self-evident, however, that $x+x$ is equivalent to x alone, whether we regard it in extent of meaning, as *all the x's added to all the x's*, which is simply *all the x's*, or in intent of meaning, as *either x or x*, which is surely x. Thus, $x+x-x$ is really 0, and not x, the required result, and it is apparent that the process of subtraction in logic is inconsistent with the self-evident Law of Unity.

194. It is probable, indeed, that Professor Boole would altogether refuse to recognise such an expression as $x+x-x$, on the ground that it does not obey the condition of his symbols that each two alternatives shall be distinct and contrary, $x+x$ not being so. It may be answered,

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that the expression has been arrived at by operations enounced as universally valid, which ought to give true results. And if it be simply said that $x+x-x$ is not *interpretable* in Professor Boole's system, it may be again answered, that when translated into its equivalent in words, the expression $x+x-x$ has a very plain meaning. It is '*either x or x, provided it be not x*,' and this, I must hold, is simply *not x*, although it ought to be x, according to the mode in which it was got.

195. In founding his system, Boole assumed that there cannot be two terms $A+B$, the same in meaning or names of the same thing; the laws of thought require nothing of the kind, and cannot require it, because among known and unknown terms, any two such as $A+B$ may prove to be names of the same thing AB. Thought merely reduces the meaning of two same terms $AB+AB$, by the Law of Unity, to be the same as that of one term AB. And when it is once known that all terms in question are contraries of each other, or naturally exclusive and distinct, then Boole's system and the whole science of numbers apply.

196. It is on this account that my objections have no bearing against Professor Boole's system as applied to the Calculus of Probabilities, so far as I can understand the subject. For it is a high advantage to that calculus to have to treat only events mutually exclusive, probabilities being then capable of simple addition and subtraction.* It seems likely, indeed, that this distinction of exclusive and unexclusive alternatives is the Gordian knot in which all the abstract logical sciences meet and are entangled.

Fourth Objection.

197. The last objection that I shall at present urge against Professor Boole's system is, that *the symbols* $\frac{1}{1}, \frac{0}{0}, \frac{0}{1}, \frac{1}{0}$, *establish for themselves no logical meaning, and only bear a meaning derived from some method of reasoning not contained in the symbolic system.* The meanings, in short, are those reached in the self-evident indirect method of the present work.

* See De Morgan's *Syllabus*, § 243.

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198. Professor Boole expressly allows, as regards one of these symbols at least, $\frac{0}{0}$, that it is not his method which gives any meaning to the symbol. It is the peculiarity of his system, that he bestows a meaning on his symbols by *interpretation*. The interpretation of $\frac{0}{0}$ is explained on pp. 89, 90, and he says, 'Although the above determination of the significance of the symbol $\frac{0}{0}$ is founded upon the examination of a particular case, yet the principle involved in the demonstration is general, and there are no circumstances under which the symbol can present itself to which the same mode of analysis is inapplicable.' Again (p. 91), 'Its actual interpretation, however, as an indefinite class symbol, cannot, I conceive, except upon the ground of analogy, be deduced from its arithmetical properties, but must be established experimentally.'

199. If I understand this aright, it simply means, that wherever a term appears in a conclusion with the symbol $\frac{0}{0}$ affixed, we may, by a mode of analysis, by some process of pure reasoning apart from the symbolic process from which $\frac{0}{0}$ emerged, ascertain that the meaning of $\frac{0}{0}$ is *some*, an *indefinite class term*. The symbol $\frac{0}{0}$ is unknown until we give it a meaning. Before, therefore, we can know what meaning to give, and be sure that this meaning is right, it seems to me we must have another distinct and intuitive system by which to get that meaning. Professor Boole's system, then, as regards the symbol $\frac{0}{0}$, is not the system bestowing certain knowledge; it is, at most, a system pointing out truths which, by another intuitive system of reasoning, we may know to be certainly true.

200. It is sufficient to show this with regard to a single symbol $\frac{0}{0}$, because the incapacity of a system, even in a single instance, proves the necessity for another system to support it. I believe that the other symbols, $\frac{1}{1}$, $\frac{1}{0}$, $\frac{0}{1}$, are open to exactly the same remarks, but from the way in which Mr. Boole treats them, involving the whole conditions of his system, it would be a lengthy matter to explain.

201. The obscure symbols $\frac{1}{1}$, $\frac{0}{0}$, $\frac{1}{0}$, $\frac{0}{1}$, have the following correspondence with the forms of the present system. $\frac{1}{1}$ appearing as the coefficient of a term means that the term is an *included subject* of the premise, so that, if

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combined with both members of the premise, it produces a self-contradictory term with neither side (§ 115).

Similarly, $\frac{0}{0}$ means that the term is an *excluded subject* of the premise, producing a self-contradictory with both sides of the premise..

And either $\frac{1}{0}$ or $\frac{0}{1}$ means that the term is a *self-contradictory or impossible term*, producing a self-contradictory term with one side only of the premise.

202. The correspondence of these obscure forms with the self-evident inferences of the present system is so close and obvious, as to suggest irresistibly that Professor Boole's operations with his abstract calculus of 1 and 0, are a mere counterpart of self-evident operations with the intelligible symbols of pure logic. Professor Boole starts from logical notions, and self-evident laws of thought; he suddenly transmutes his formulæ into obscure mathematical counterparts, and after various intricate manœuvres, arrives at certain forms, corresponding to forms arrived at directly and intuitively by ordinary or Pure Logic—by that analysis, from which the interpretation of his symbols was reached and proved. And by this interpretation he transfers the meaning and force of pure logical conclusions to obscure forms, which, if they have meaning, have certainly no demonstrative force of themselves. Boole's system is like the shadow, the ghost, the reflected image of logic, seen among the derivatives of logic.

203. Supposing it prove true that Professor Boole's Calculus of 1 and 0 has no real logical force and meaning, it cannot be denied that there is still something highly remarkable, something highly mysterious in the fact, that logical forms can be turned into numeral forms, and while treated as numbers, still possess formal logical truth. It proves that there is a certain identity of logical and numerical reasoning. Logic and mathematics are certainly not independent. And the clue to their connection seems to consist in distinct logical terms forming the units of mathematics.

204. Things as they appear to us in the reality of nature, are clothed in inexhaustible attributes, set as it were in a frame of time and space. By our mental powers we abstract first time, then space, and then attribute after attri-

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bute, until we can finally think of things as abstract units deprived of all attributes, and only retaining the original logical condition of things, that each is distinct from others. In logic we argue upon things as same and one, in number we reason upon them as distinct and many.

205. Supposing it be ultimately allowed that Professor Boole's calculus of 1 and 0 is not really logic at all; that his system is founded upon one condition, that of exclusive terms, which does not belong to thought in general, but only numerical thought; and that it ignored one law of logic, the Law of Unity, which really distinguishes a logical from a numerical system—these errors scarcely detract from the beauty and originality of the views he laid open. Logic, after his work, is to logic before his work, as mathematics with equations of any degree are to mathematics with equations of one or two degrees. He generalised logic so that it became possible to obtain any true inference from premises of any degree of complexity, and the work I have attempted has been little more than to translate his forms into processes of self-evident meaning and force.

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